EIDMA MINICOURSE ON STRUCTURAL GRAPH THEORY

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INTRODUCTORY PROBLEMS

1. Let $\mathcal{T}$ be a family of subtrees of a tree T and let k be an integer. Prove that either $\mathcal{T}$ has k pairwise disjoint members, or T has a set of strictly fewer than k vertices that intersects each member of $\mathcal{T}$. In particular, if every two trees in $\mathcal{T}$ have a vertex in common, then they all have a vertex in common.

2. A graph G is *series-parallel* if it contains no subgraph isomorphic to a subdivision of K_4. Prove that no series-parallel graph is 3–connected.

3. Prove that a graph is series-parallel if and only if it can be reduced to the null graph by repeatedly deleting loops, parallel edges and vertices of degree at most one, and suppressing vertices of degree two (that is, contracting one of the incident edges).

4. Prove that every loopless series-parallel graph is 3-colorable.

5. Design a linear-time algorithm to test if a graph is series-parallel.
Warning. This is not as trivial as it may seem. The graph is represented by its adjacency list. Can you delete an edge in constant time?

6. Prove that a minimal (with respect to inclusion) vertex cut in a chordal graph is a clique.

7. Prove that every chordal graph has a simplicial vertex (that is, a vertex whose neighborhood is a clique).

8. Prove that for every integer t there exists an integer n such that every 2-connected graph on at least n vertices has either a cycle of length at least t or a $K_{2,t}$ minor.