EIDMA MINICOURSE ON STRUCTURAL GRAPH THEORY

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LECTURE 1: Planar graphs and graphs on surfaces

Topics: Review of basic properties of planar graphs, Kuratowski's theorem and the uniqueness of planar embeddings. Brief discussion of Steinitz's theorem, circle packing, Schnyder's theorem and drawing on a grid, and geometric graphs. $Y\Delta$ transformations and applications. Planar separators. The emergence of planarity in graph structure theory, the two-paths problem. Graphs on surfaces, representativity (face-width).

Recommended reading: [D, Chapter 4] for planar graphs, [MT] or an algebraic topology textbook (such as Massey's book) for the classification of surfaces.

Exercises

1. Let G be a simple 3-connected non-planar graph. Prove that either G is isomorphic to K_5 or G has a subgraph isomorphic to a subdivision of $K_{3,3}$.

2. Prove that for every planar graph G there exists an integer k such that G is isomorphic to a minor of the $k \times k$ grid.

Let G be a graph, and let v be a vertex of degree three with distinct neighbors v_1, v_2, v_3. A $Y\Delta$ *transformation* is the operation of deleting v and adding the edges of a triangle with vertex-set $\{v_1, v_2, v_3\}$. The reverse operation is called a ΔY *transformation*.

A graph G is $Y\Delta$-*reducible* if it can be reduced to the null graph by a series of the following operations:

- $Y\Delta$ and ΔY transformation,
- deletion of a loop, a parallel edge or a vertex of degree at most one,
- suppressing a vertex of degree two.

(See preliminary exercises.)

3. Prove that the $k \times k$ grid is $Y\Delta$-reducible for every integer $k \geq 0$.

4. Prove that if H is a minor of G and G is $Y\Delta$-reducible, then so is H.

5. Prove that every planar graph is $Y\Delta$-reducible.

6. Prove that K_6 and the Petersen graph are not $Y\Delta$-reducible.

7. Construct a graph G such that G is not $Y\Delta$-reducible, and yet it has a vertex v such that $G \backslash v$ is planar.