EIDMA MINICOURSE ON STRUCTUAL GRAPH THEORY
Robin Thomas
LECTURE 3: Tree-decompositons of graphs II.

Topics: Branch-width and algorithms. The graph minor theorem and its applications.

Recommended reading: [D, Chapter 12], [1] for branch-width, [2] for algorithms, and [3] for the graph minor theorem.

Exercises

1. Prove that for $n \geq 3$ the complete graph on n vertices has branch-width $\lceil 2n/3 \rceil$.

2. Prove that if a graph G has a cycle, then $\text{bw}(G) \leq \text{tw}(G) + 1 \leq 3\text{bw}(G)/2$.

3. Prove that both inequalities in the above exercise are tight for infinitely many n.

4. Prove that a graph G has branch-width at most two if and only if it is series-parallel.

5. Prove that a planar graph of tree-width at least $10k$ has a $k \times k$ grid minor.

References

1. N. Robertson and P. D. Seymour, Graph Minors X. Obstructions to tree-decomposition, *J. Combin. Theory Ser. B* **52** (1991), 153–190.
2. N. Robertson and P. D. Seymour, Graph Minors XIII. The disjoint paths problem, *J. Combin. Theory Ser. B* **63** (1995), 65–110.
3. N. Robertson and P. D. Seymour, Graph Minors XVI. Excluding a non-planar graph, *J. Combin. Theory Ser. B* **89** (2003), 43–76.