EIDMA MINICOURSE ON STRUCTURAL GRAPH THEORY

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LECTURE 6: Beyond the Four-Color Theorem

Topics: Unique coloring, edge-coloring, packing T-joins, nowhere-zero flows, cycle double covers, Hadwiger's conjecture. Jorgensen's conjecture. Extremal problems for minors. Linkages.

Recommended reading: [D, Chapter 6] or [1] for nowhere-zero flows.

Exercises

1. Let $c : E(G) \to \{1, 2, 3\}$ be a 3-edge-coloring of a 3-regular plane graph G. We say that a vertex v is *positive* if the colors $1, 2, 3$ appear around v in that clockwise order, and we say that v is *negative* otherwise. The *sign* of c is the product of the signs of all the vertices. Prove that every two 3-edge-colorings of G have the same sign.

2. Let $n \geq 4$. Prove that every simple graph G on n vertices with no K_6 minor has at most $4n-10$ edges. Prove that the bound is best possible for every $n \geq 4$.

3. Prove that a.e. graph in the random graph model $\mathcal{G}(n, 1/2)$ satisfies Hadwiger's conjecture.

4. Prove that a.e. graph in the random graph model $\mathcal{G}(n, 1/2)$ is a counterexample to Hajos' conjecture (that is, $t < \chi(G)$ for every t such that G has a subgraph isomorphic to a subdivision of K_t.

References

1. P. D. Seymour, Nowhere-zero flows, in: Handbook of combinatorics, (eds. Graham, Grötschel, Lovász), North-Holland, 1995.