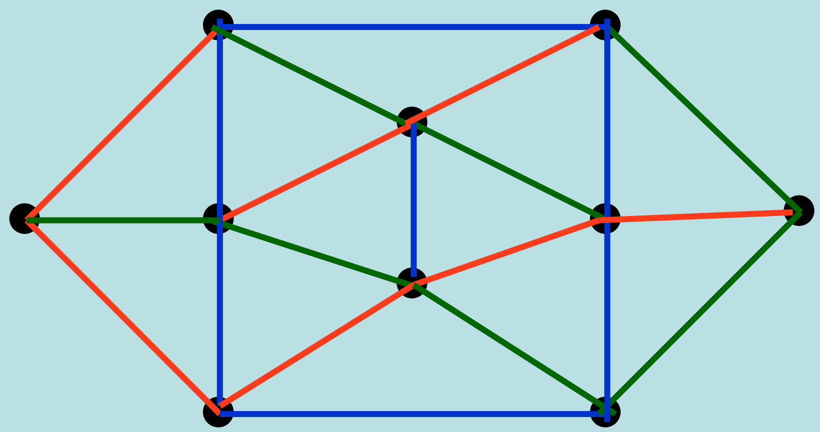
A tri-coloring of a planar triangulation G is a mapping $E(G) \rightarrow \{1,2,3\}$ such that every facial triangle is 3-colored.



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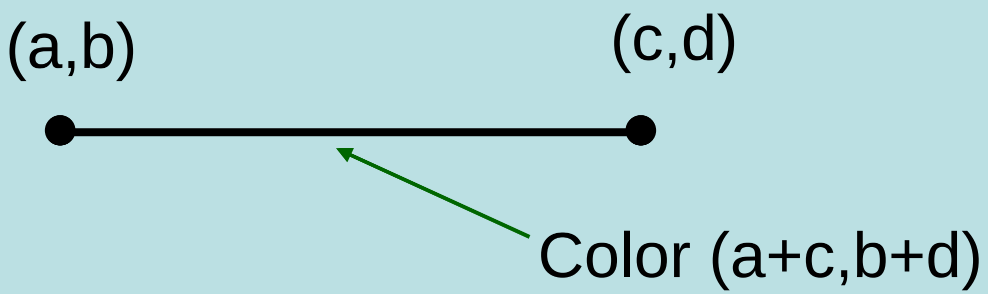
There is a 1-1 correspondence between 4-colorings and tri-colorings:



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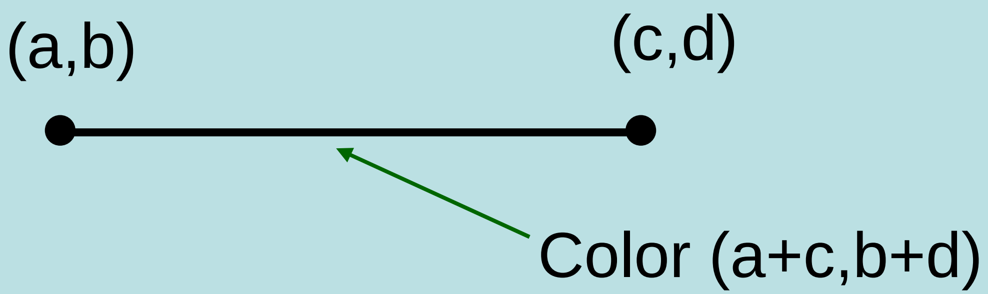

We will work with tri-colorings instead.

CONSISTENCY

Let $\mathcal{C}$ be a set of tri-colorings of a cycle *R*. We say $\mathcal{C}$ is realizable if there exists a near-triangulation *G* with its outer face bounded by *R* such that $\mathcal{C}$ is precisely the set of tri-colorings that extend to a tri-coloring of *G*.

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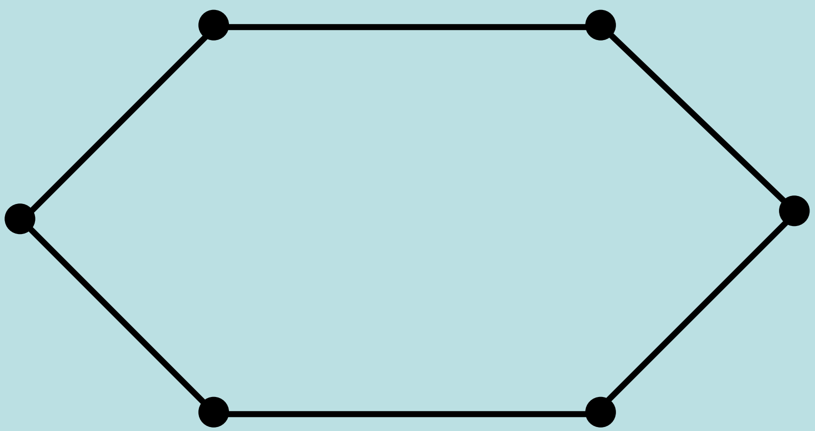

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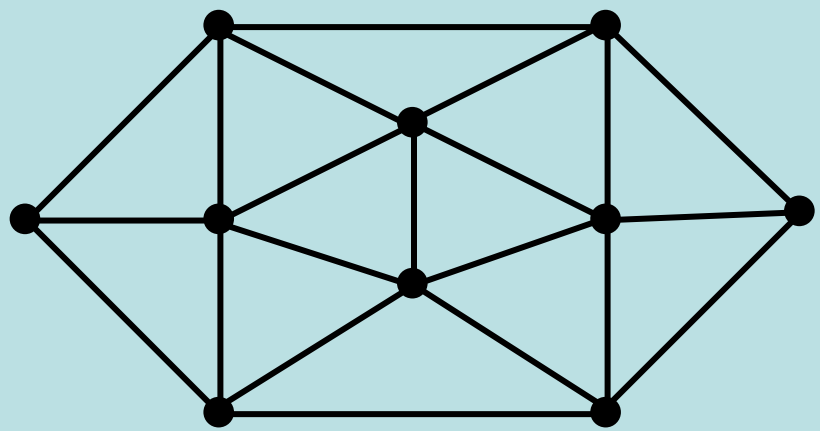
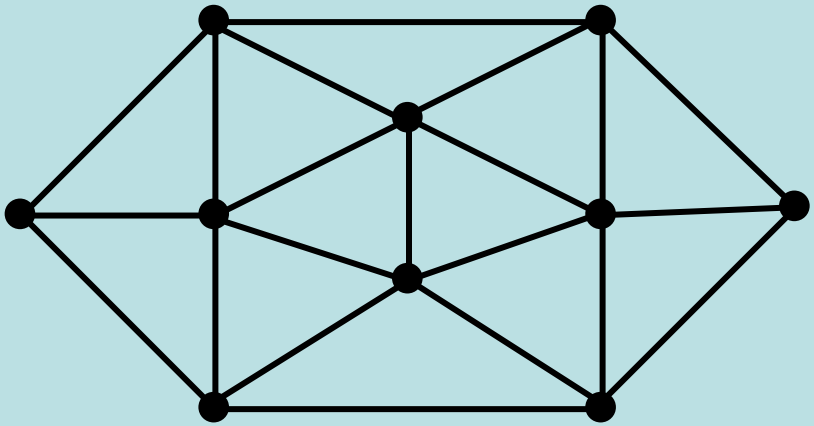


If $\mathcal{C}$ is realizable, then for every *c* in $\mathcal{C}$ and every pair of colors *a, b* there exists a planar matching *M* of edges of that color (“Kempe chain”) such that if we swap *a* and *b* on any subset of *M*, the new coloring belongs to $\mathcal{C}$.

The latter property is consistency.



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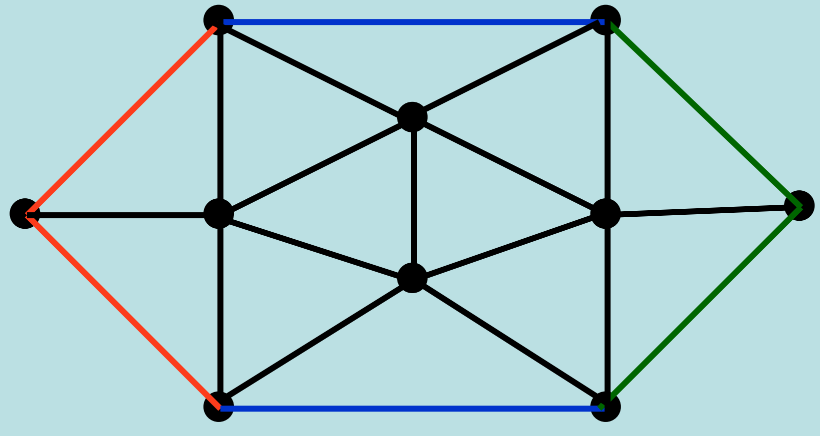

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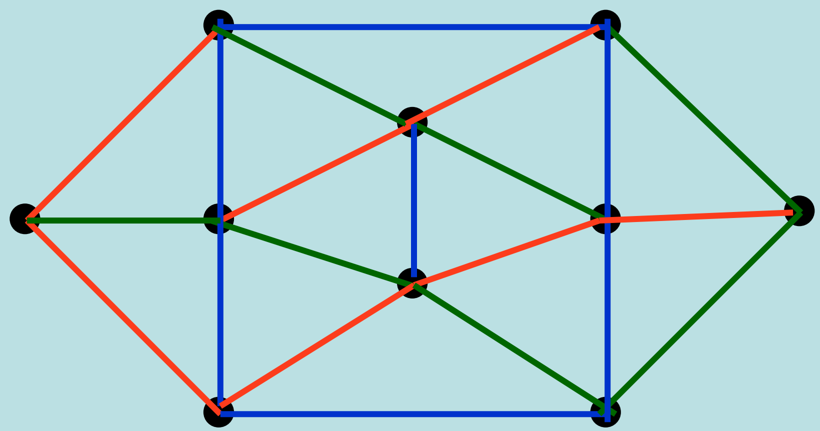
Example: a=red, b=green



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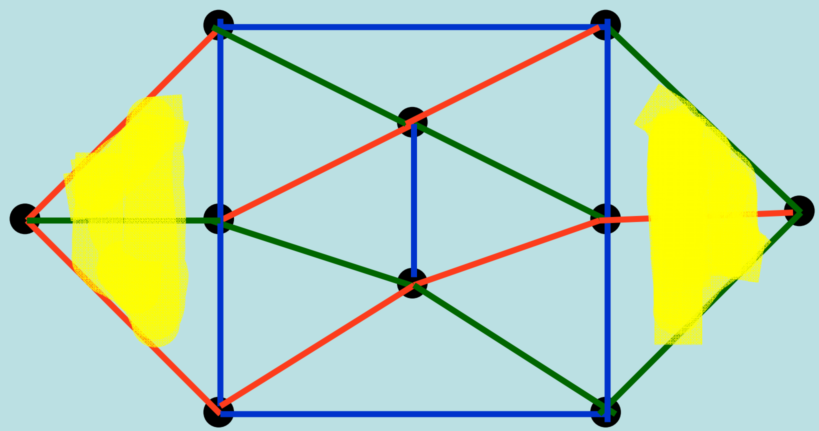

We need a stronger property, introduced by A. Bernhart and Cohen. It counts colorings compatible with given matching rather than noting whether they exist.

Let i=0,1,2. A tri-coloring c of R is i-compatible with a signed matching M if

- M matches edges not colored i
- positively matched edges colored the same
- negatively matched edges colored differently

c is *i*-compatible with a signed matching *M* if

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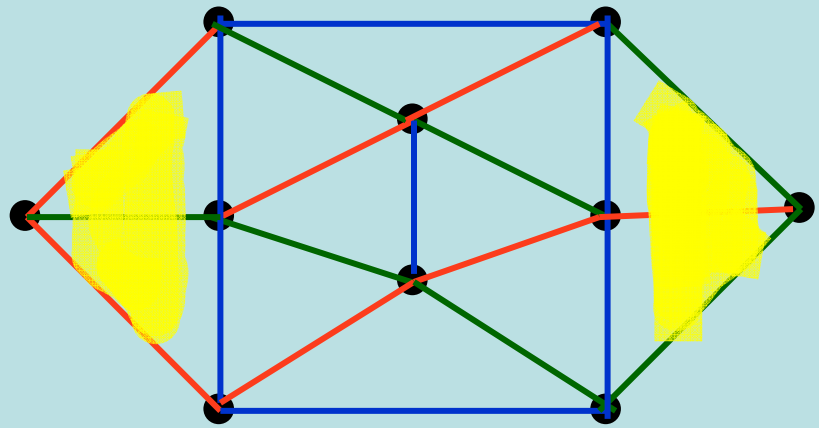

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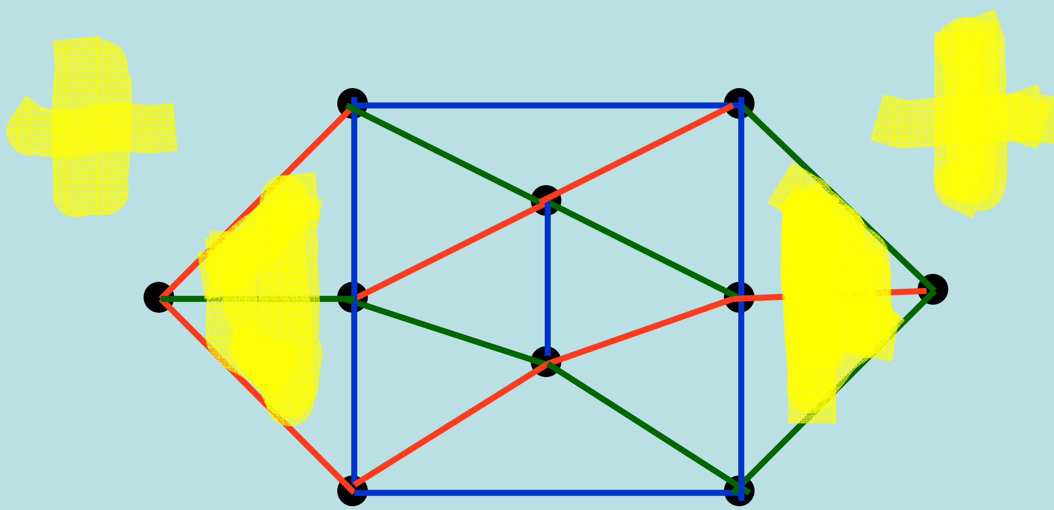

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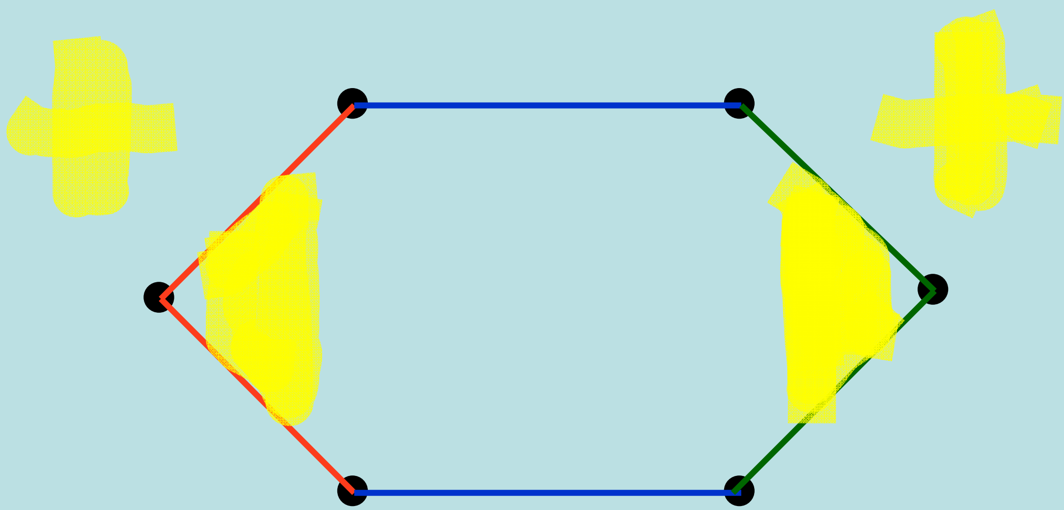

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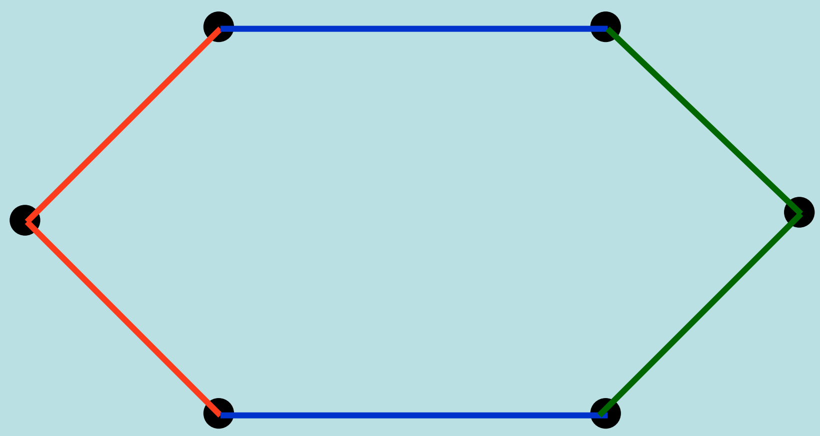

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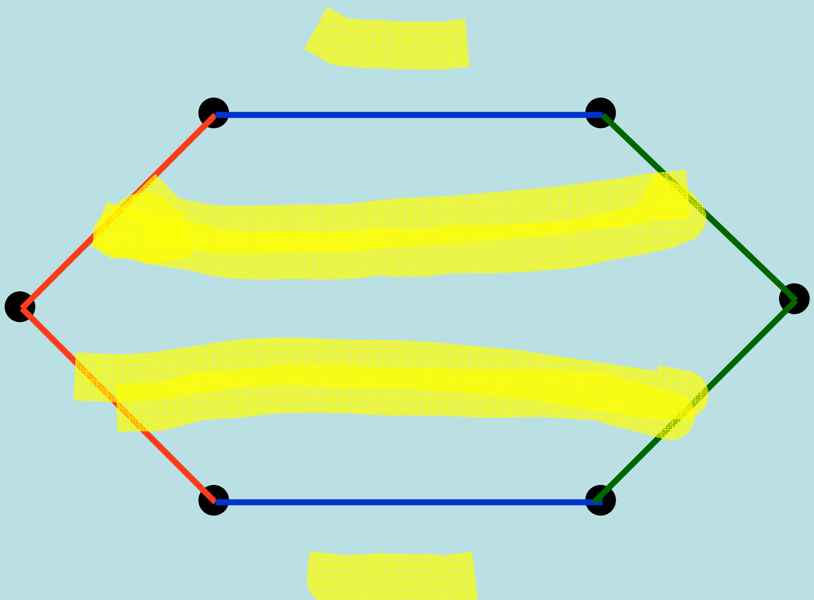


A set of colorings $\mathcal{C}$ of a cycle R is block-count (BC) consistent if for every planar signed matching M there exists an integral variable $x_M \geq 0$ such that for every coloring c in C

$$\Sigma(x_M : M, c \text{ are } i\text{-compatible})$$

is independent of $i=0,1,2$.

Facts: Realizable ⇨BC-consistent

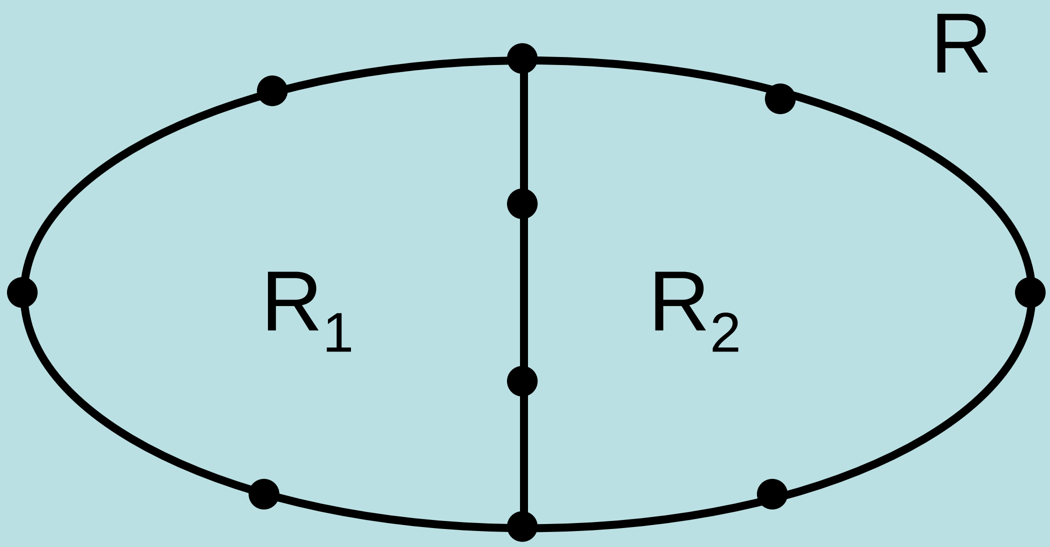
Union of BC-consistent sets is BC-consistent

For a configuration K let $\mathcal{J}(K)$ denote the set of all tri-colorings of the ring of K that extend into K. Let $\mathcal{E}(K)$ denote the maximal BC-consistent subset of Ω-$\mathcal{J}(K)$.

A configuration K is D-reducible if $\mathcal{E}(K)$ is empty.

A configuration is C-reducible if there exists a smaller configuration K' such that $\mathcal{E}(K)$ is disjoint from $\mathcal{J}(K')$.

Let $\mathcal{C}_1$, $\mathcal{C}_2$ be consistent sets on rings R_1, R_2. The product $\mathcal{C}_1 \otimes \mathcal{C}_2$ is the consistent set on R of all colorings c such that there exist $c_i \in \mathcal{C}_i$ such that c, c_1, c_2 agree on shared paths.



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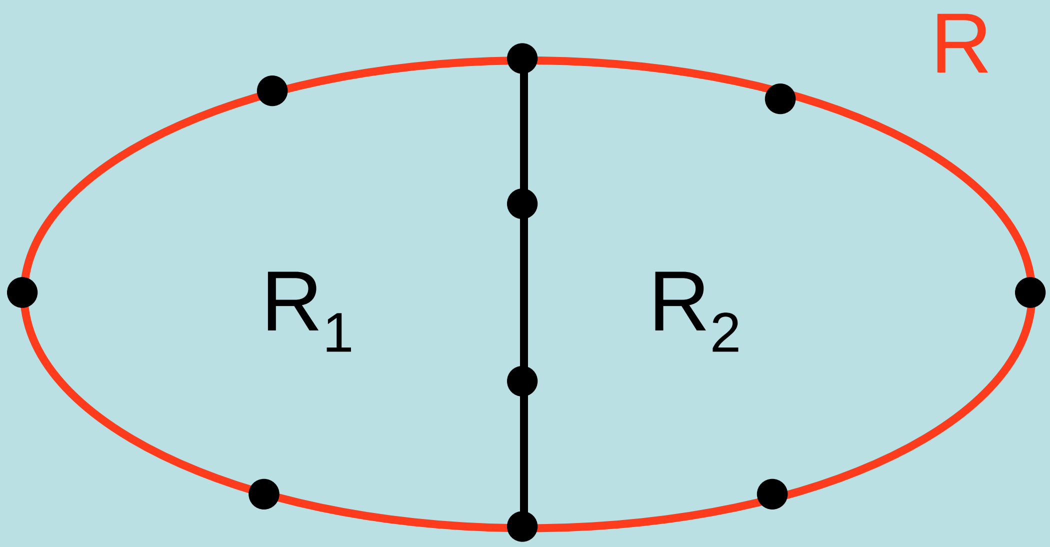

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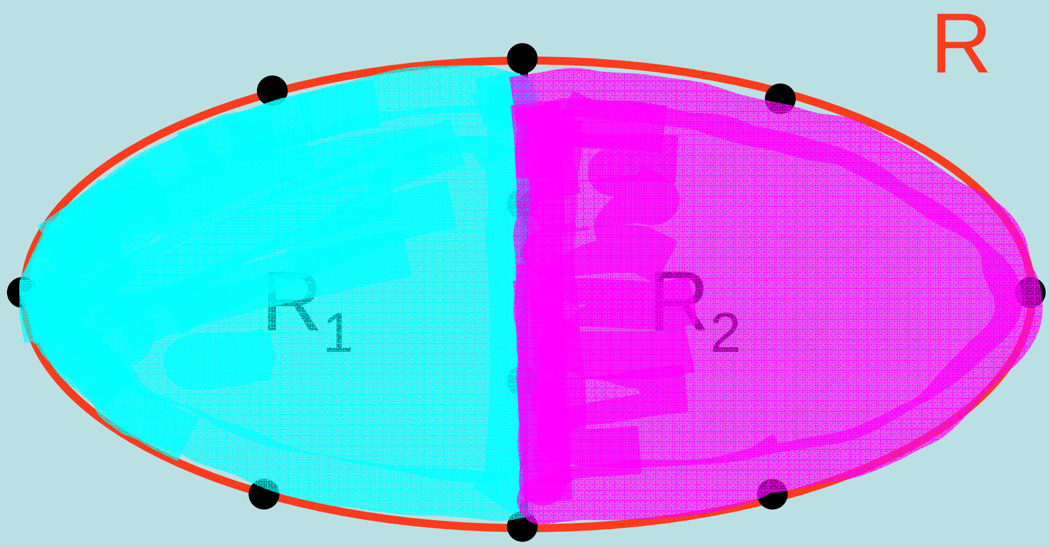
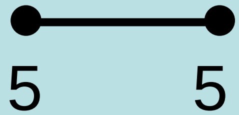
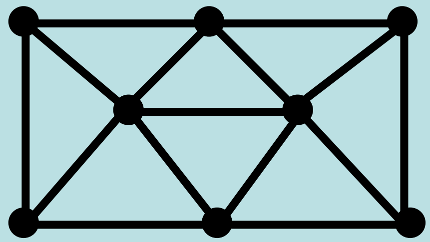
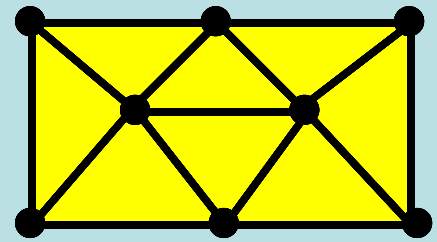
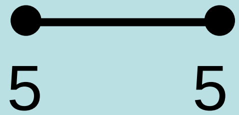
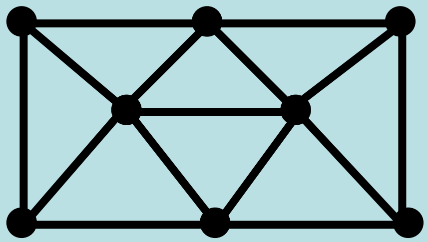


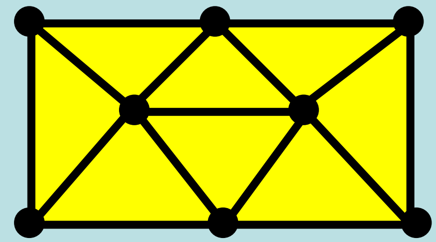
Notation: If K= 

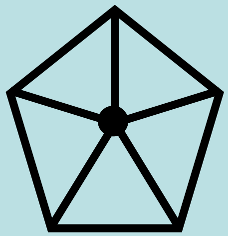
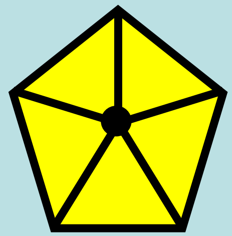
$\mathcal{J}(K)=$ 

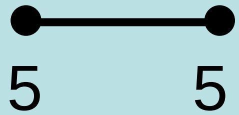
$\mathcal{E}(K)=$ 

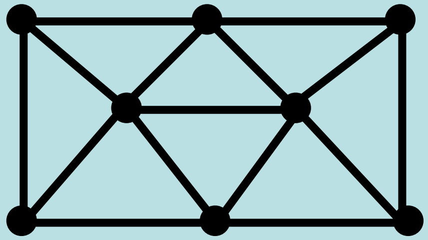
Notation: If K= 

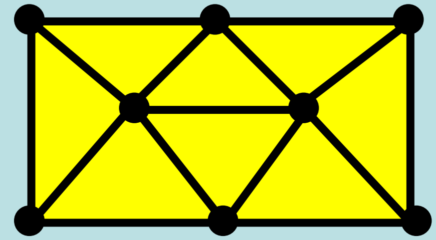
$\mathcal{J}(K)=$ 

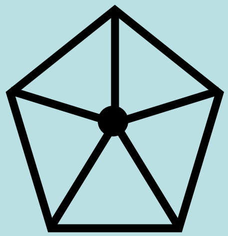
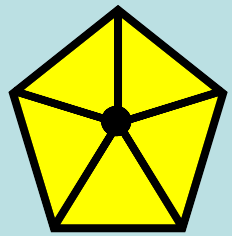
$\mathcal{E}(K)=$ 

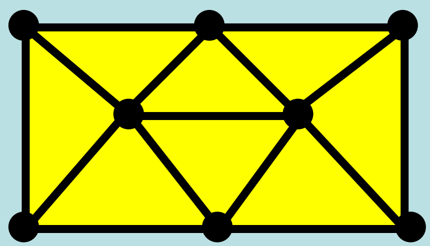
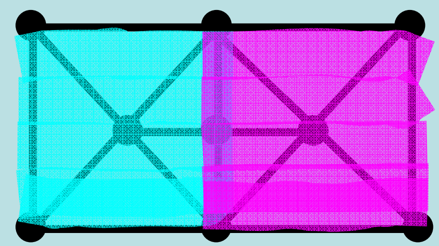
Fact:  $\cup$  $= \Omega$

Notation: If K= 

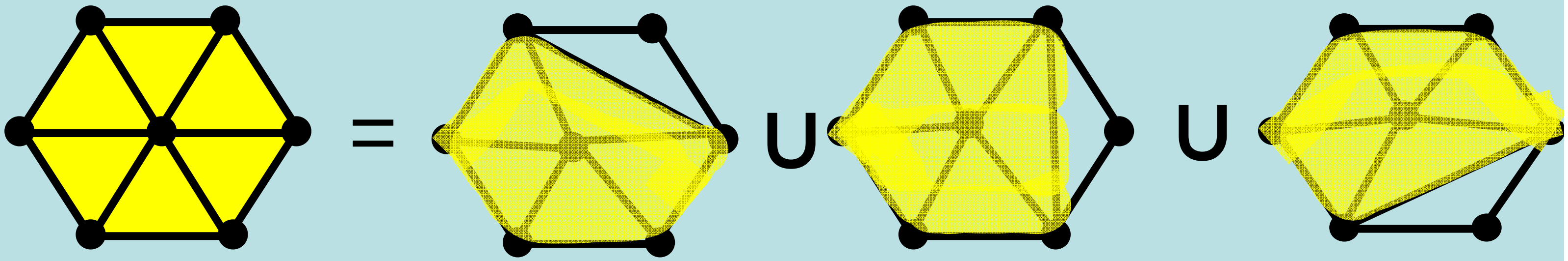
$\mathcal{I}$(K)= 

$\mathcal{E}$(K)= 

Fact:  $\cup$  $= \Omega$

THM (Bernhart)  = 

THM
Allaire



THM
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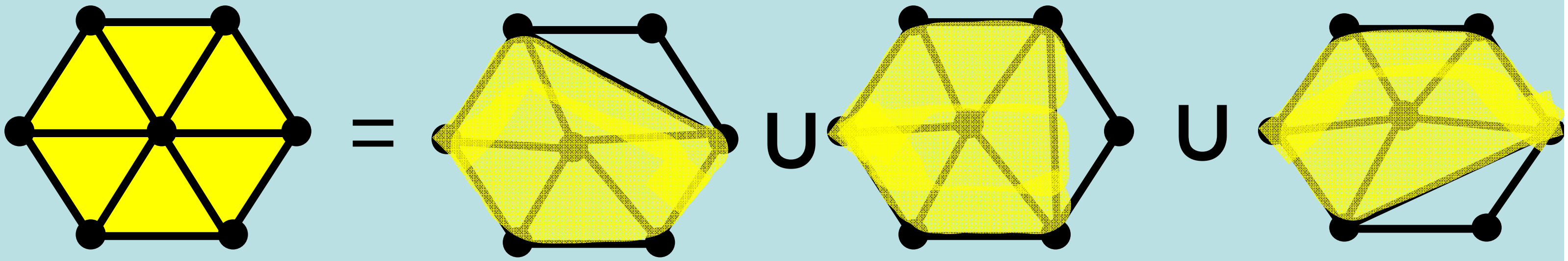
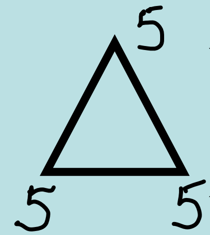
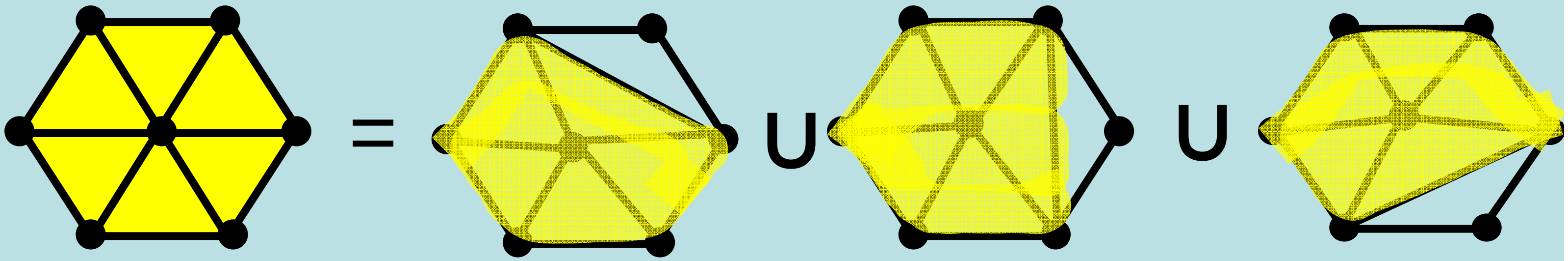
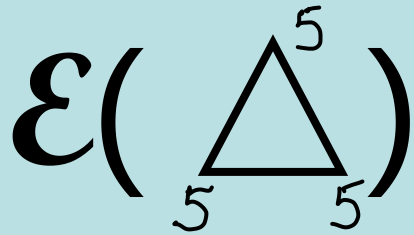


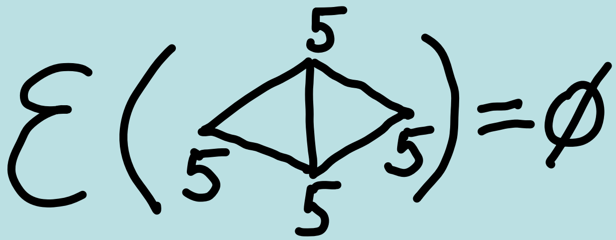
No description for $\mathcal{E}($  $)$.

THM
Allaire

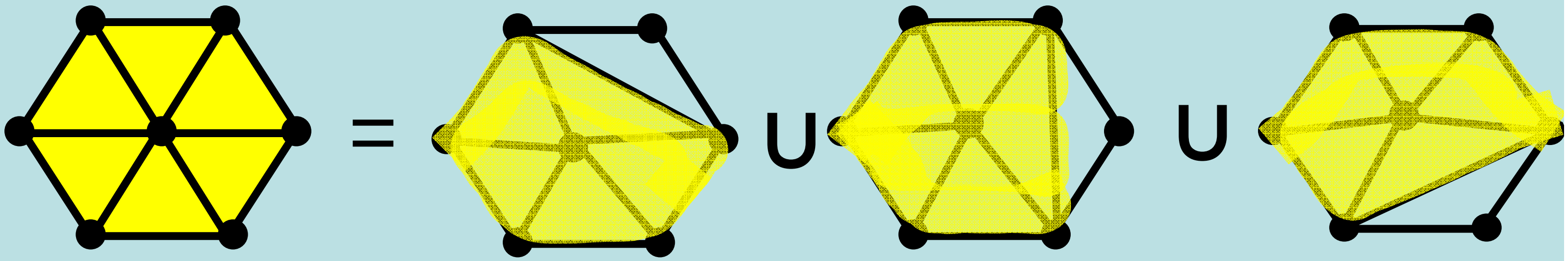


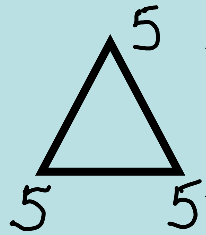
No description for .

THM
Birkhoff

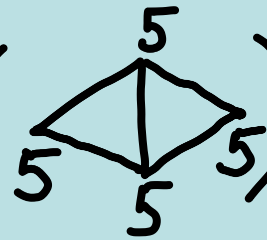


THM
Allaire

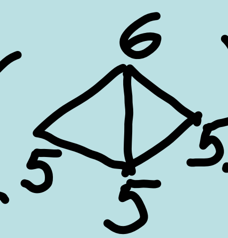


No description for $\mathcal{E}($  $)$.

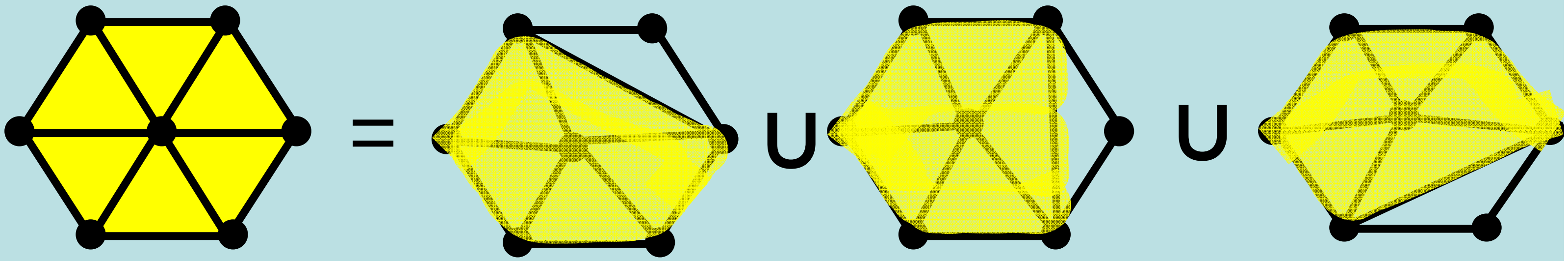
THM
Birkhoff

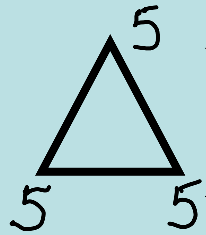
$\mathcal{E}($  $) = \emptyset$

THM
Franklin

$\mathcal{E}($  $) = \emptyset$

THM
Allaire



No description for $\mathcal{E}($  $)$.

THM
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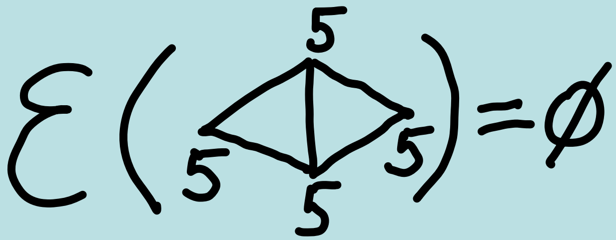
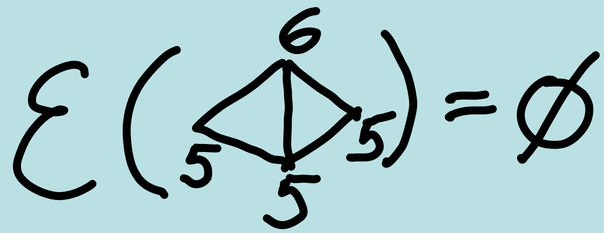
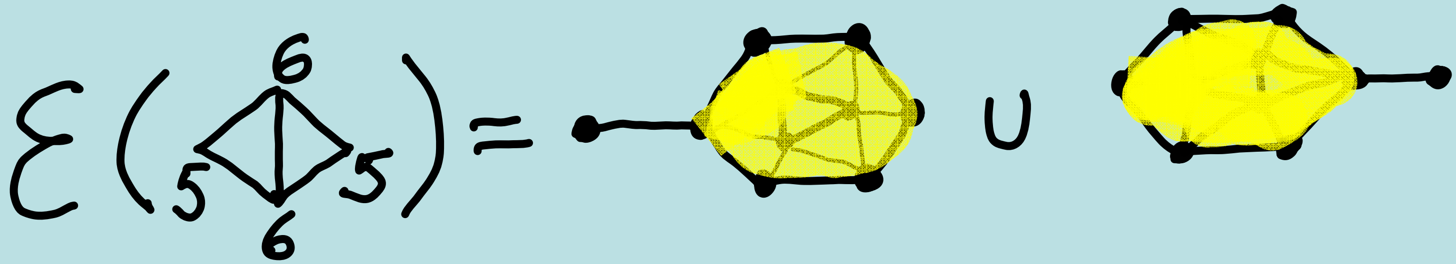


THM
Franklin



THM
Rolle



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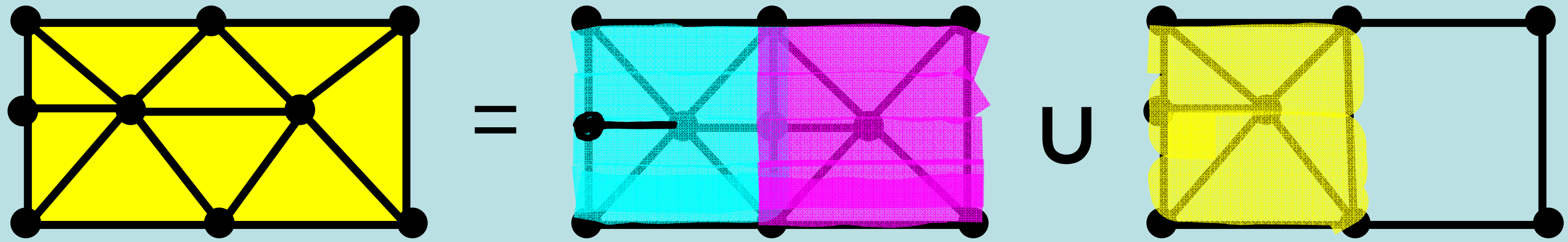
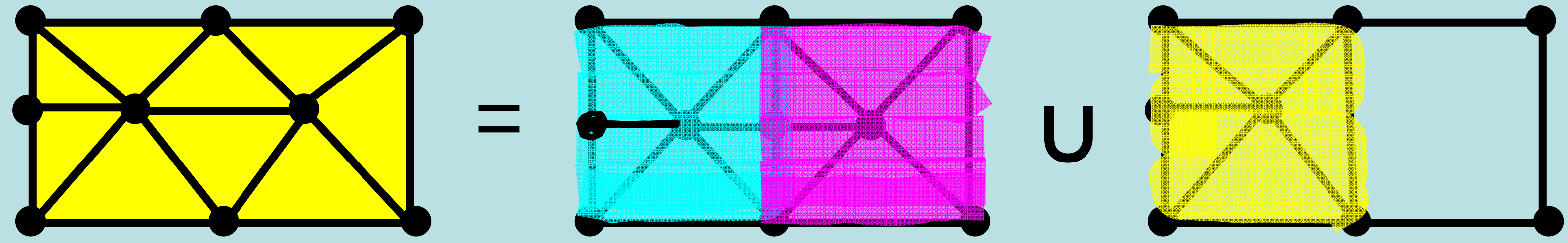


THM



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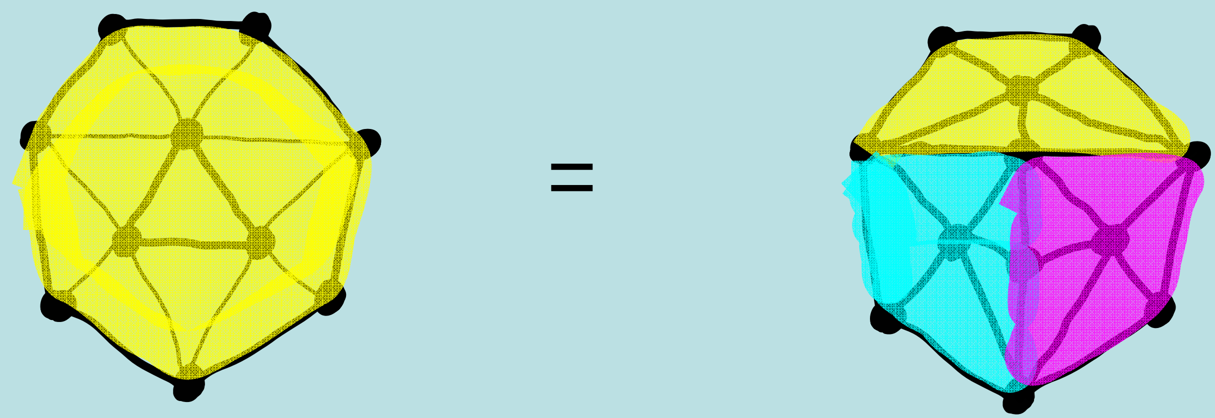

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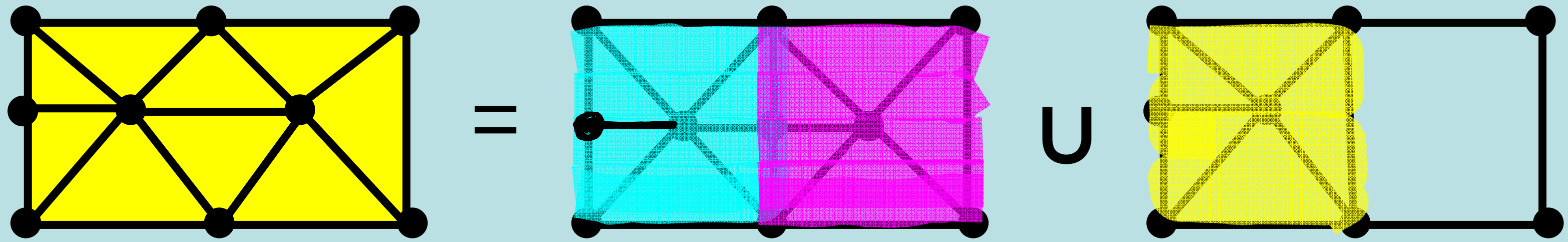

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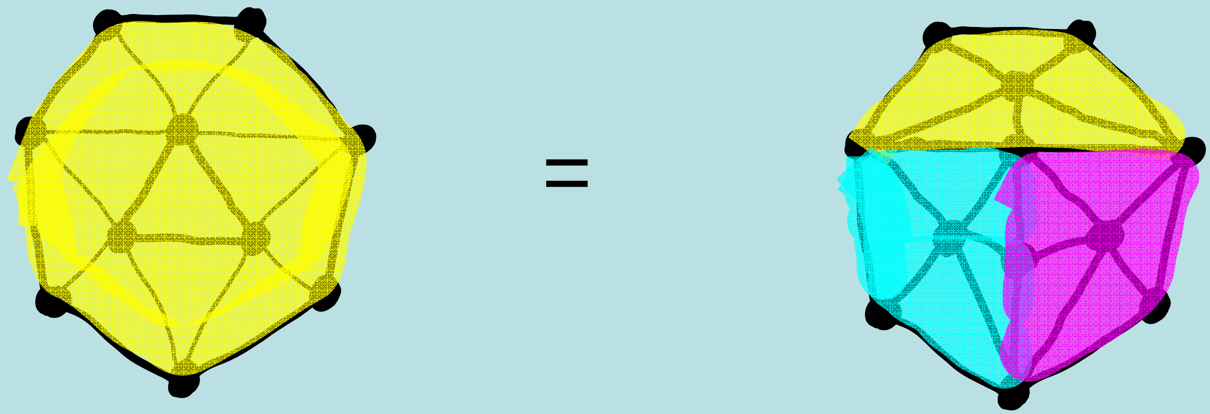

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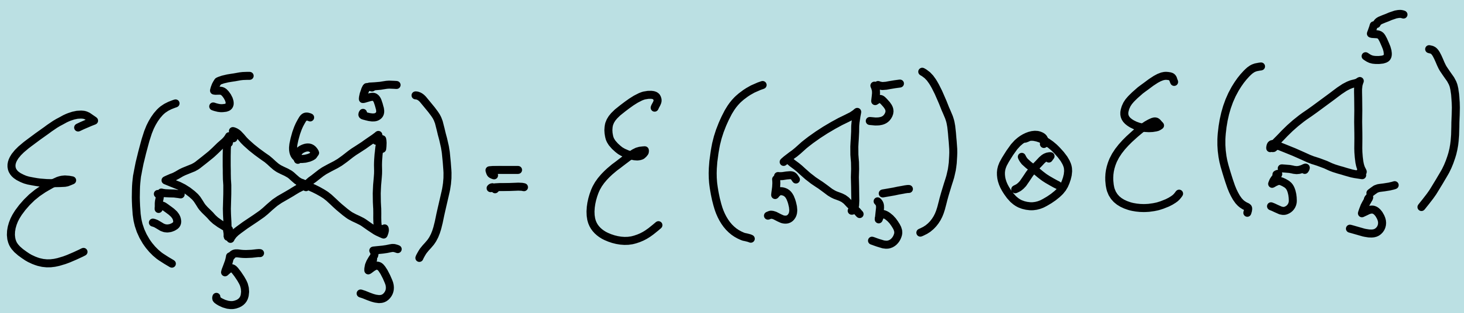
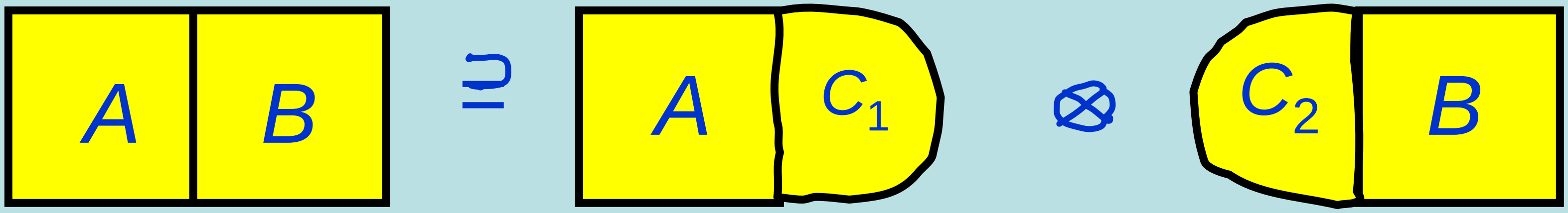


THM If A,B,C_1,C_2 are BC-consistent and $C_1 \cup C_2=\Omega$, then $\mathcal{E}(A\otimes B)\supseteq\mathcal{E}(A\otimes C_1)\otimes\mathcal{E}(C_2\otimes B)$.



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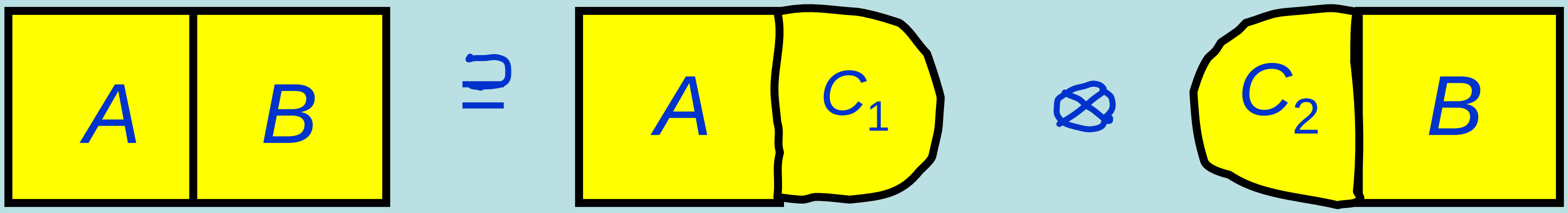

Example

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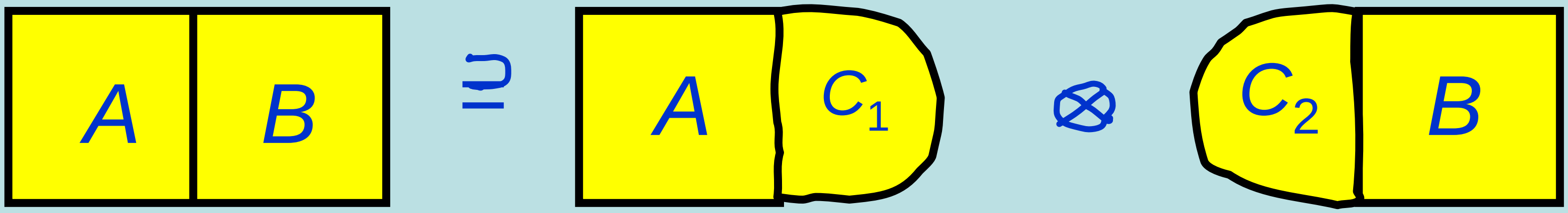
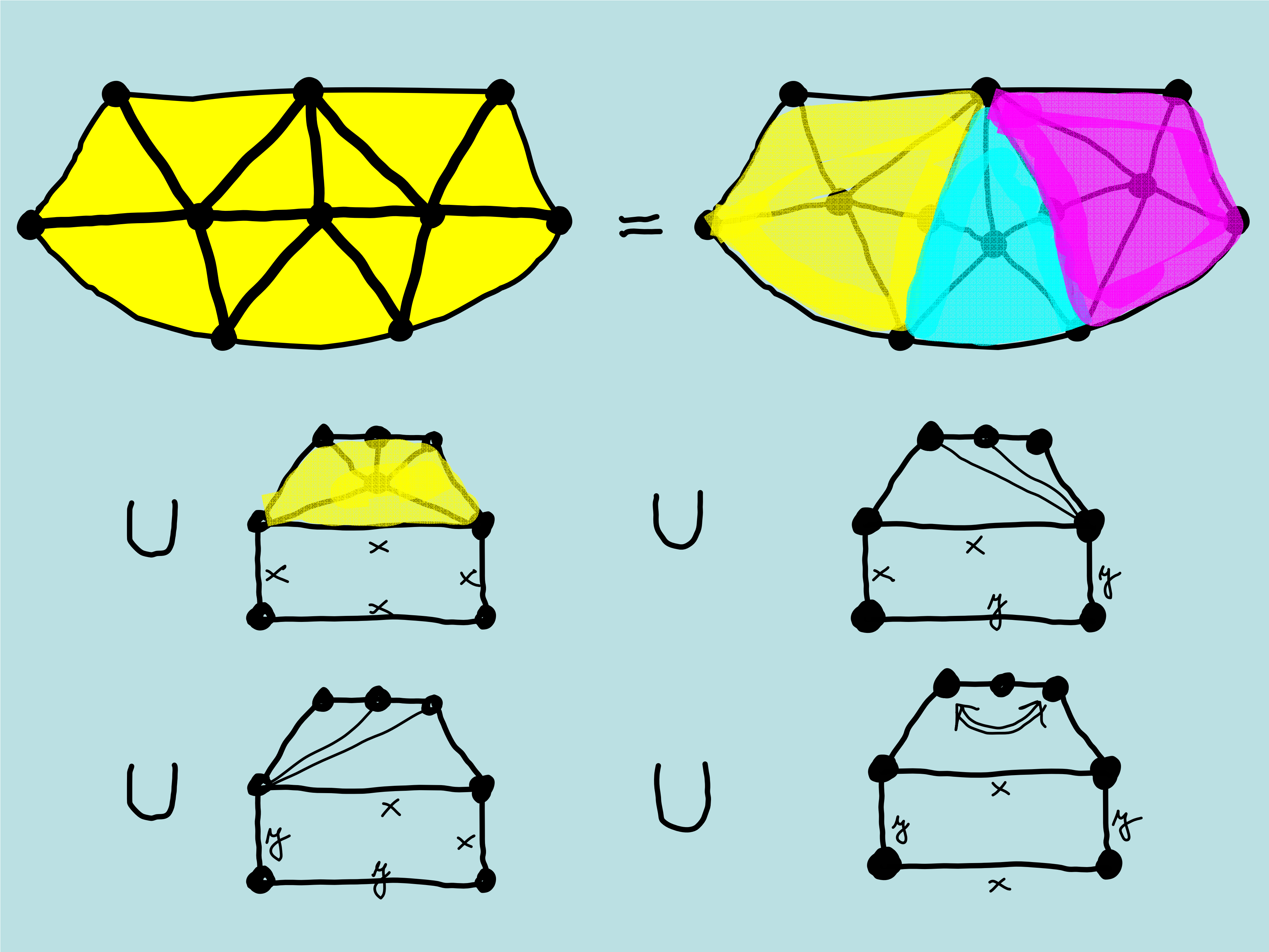


Example

In general, equality does not hold.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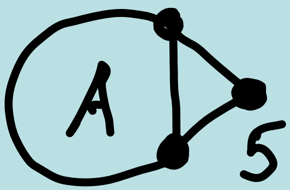
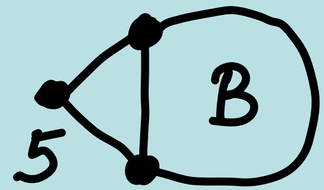
CONJECTURE (Düre, Heesch, Miehe) If

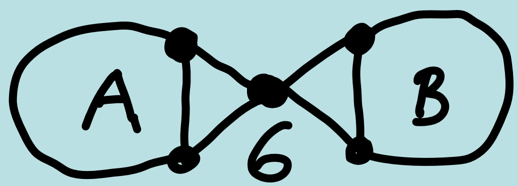


and



are D-reducible,

then



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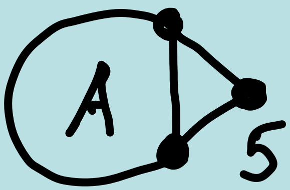
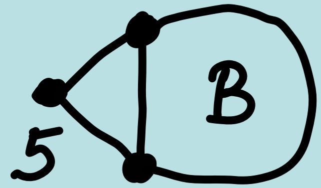


and



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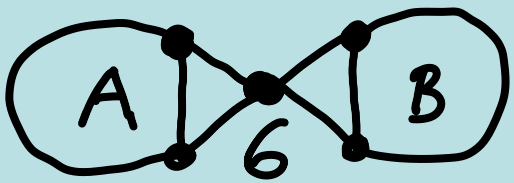
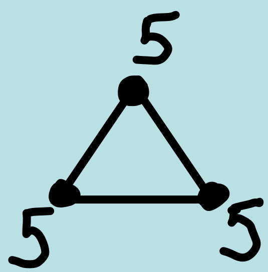
then



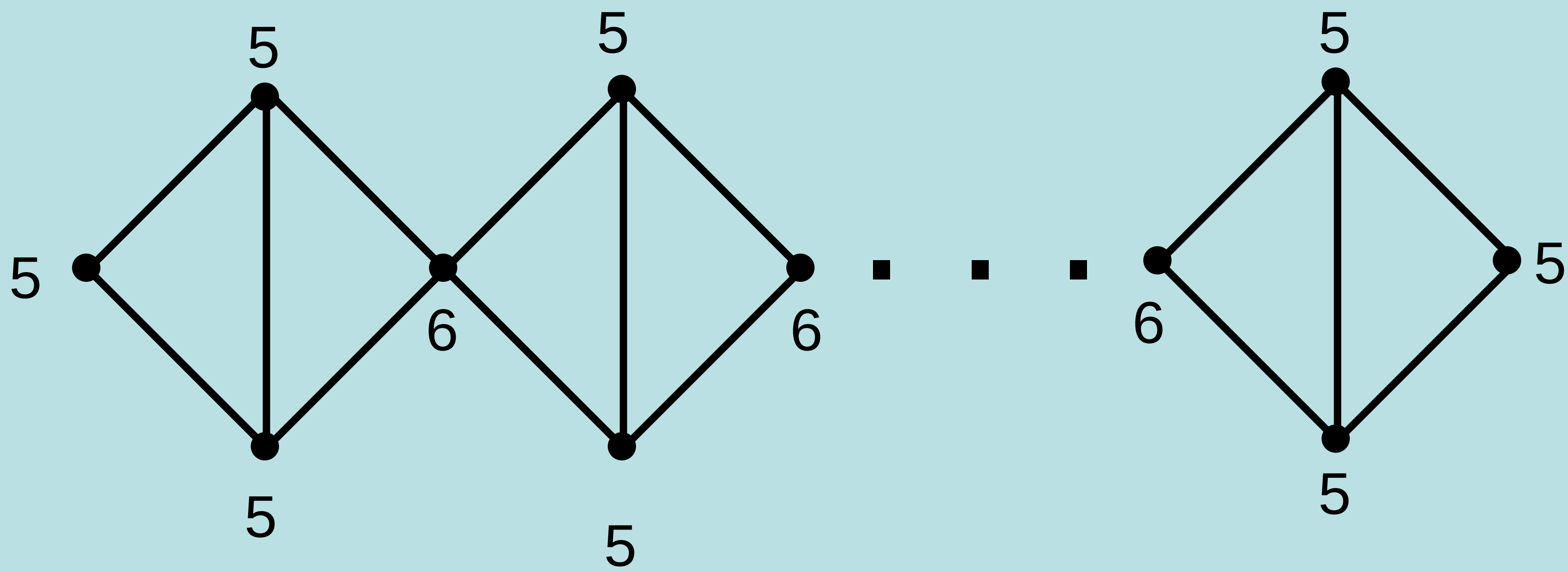
is D-reducible.

Implies that



cannot appear in a minimal counterexample.

THM The following configuration is D-reducible



The cycle double cover conjecture

Every 2-connected graph has a circulation double cover

The cycle double cover conjecture

Every 2-connected graph has a circulation double cover

union of disjoint cycles

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Enough to prove for cubic graphs

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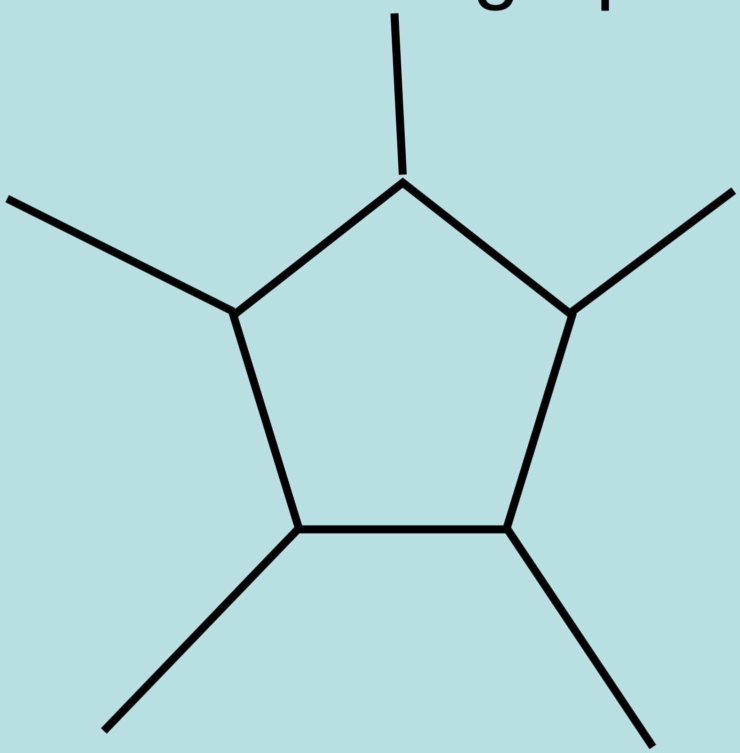

Take an induced cycle C and delete $E(C)$. Get pendant edges. A labeling assigns to each pendant edge a pair of distinct labels such that each label occurs even number of times.

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Consistency defined similarly as in 4CT, except that matchings are not necessarily planar.

Take an induced cycle *C* and delete *E*(*C*). Get pendant edges. A labeling assigns to each pendant edge a pair of distinct labels such that each label occurs even number of times.

A labeling is feasible if it is induced by a quasi-circulation of *G*-*E*(*C*).

Consistency defined similarly as in 4CT, except that matchings are not necessarily planar.

THM (Huck) Cycles of length <10 are D-reducible

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THM (Huck) Cycles of length <10 are D-reducible

COR Min counterexample to CDC has girth >9

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THM (Huck) Cycles of length <10 are D-reducible

COR Min counterexample to CDC has girth >9

CONJECTURE Every cycle is D-reducible.

The 5-flow conjecture Every 2-connected graph has a 5-flow.

Enough to prove for cubic graphs.

Kochol's reducibility method: