LARGE 6-CONNECTED GRAPHS WITH NO K_6 MINOR

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Joint work with

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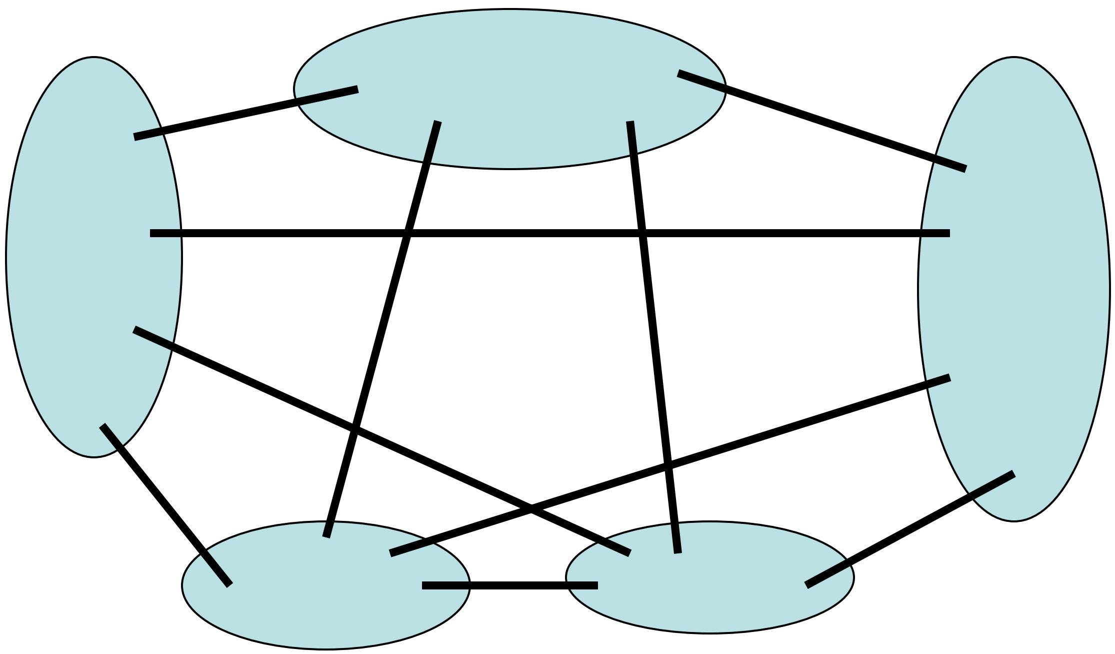
- Serguei Norine
- Paul Wollan

- A minor of G is obtained by taking subgraphs and contracting edges.
- Preserves planarity and other properties.
- G has an H minor ($H \leq_m G$) if G has a minor isomorphic to H.
- A K_5 minor:



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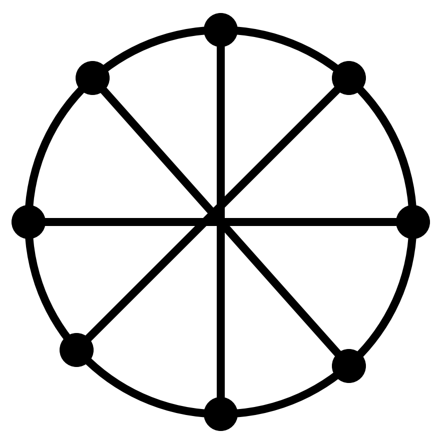

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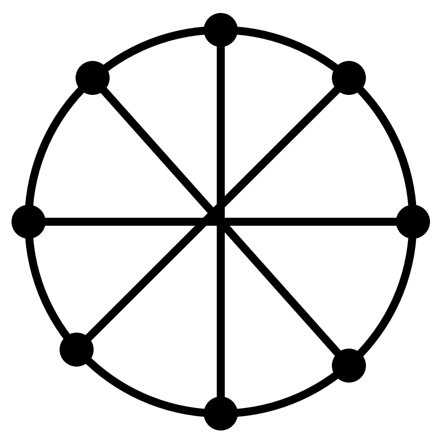

Graphs with no K_6

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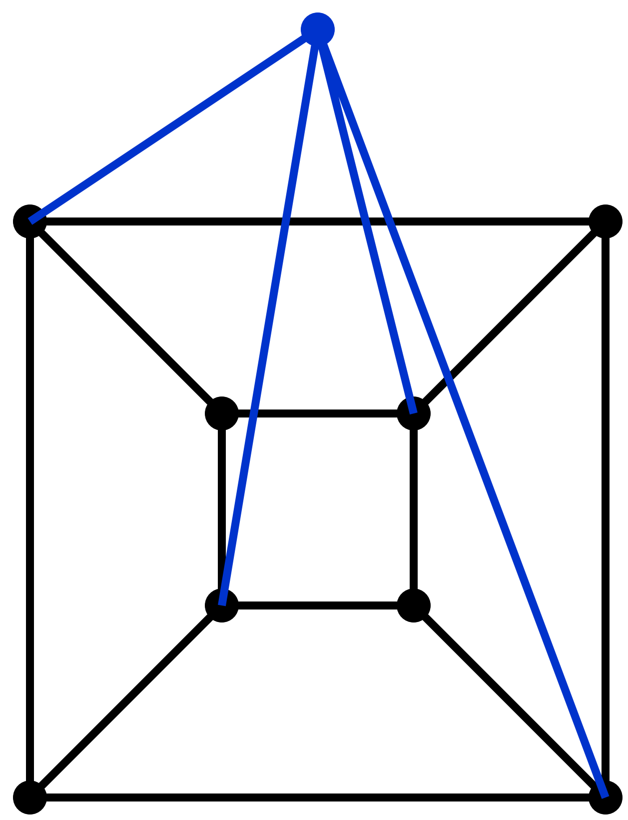

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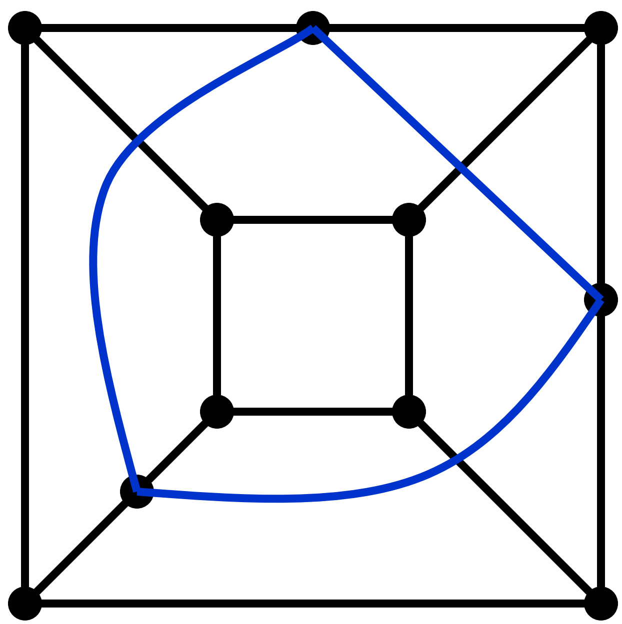

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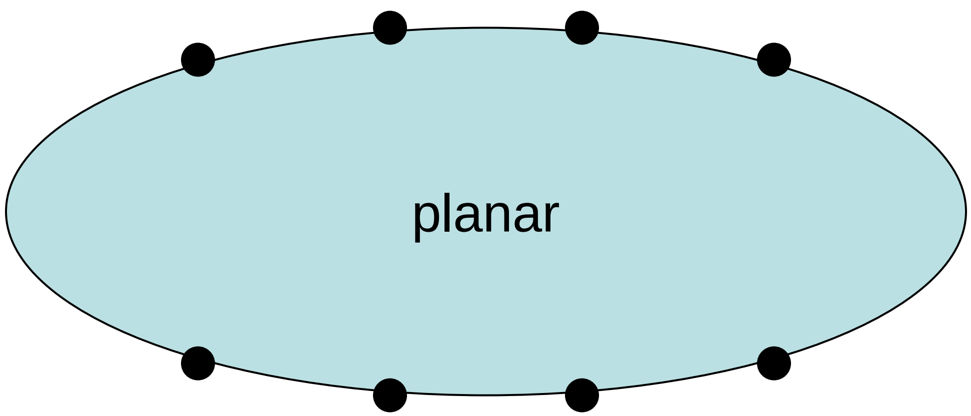

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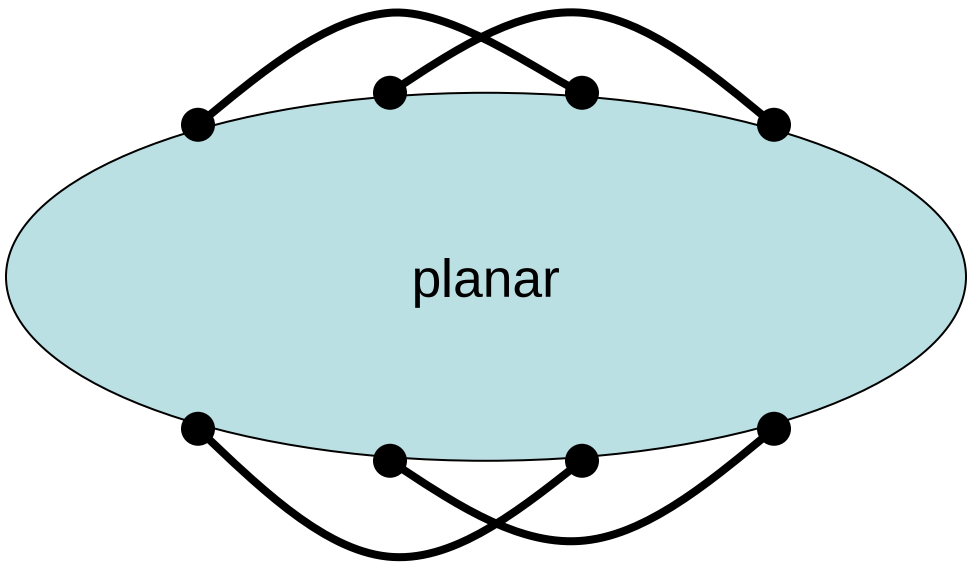

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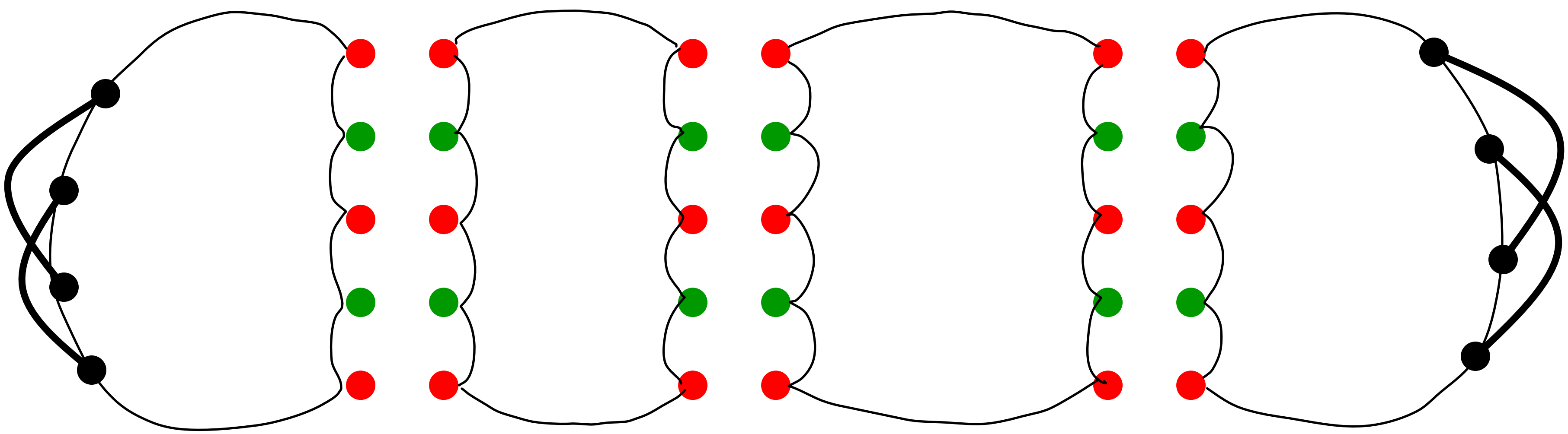

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- $G \not\geq K_9 \Rightarrow |E(G)| \leq 7n-28$, unless…. (Song, RT)

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Why easier for big graphs? Our method affords

Principle (P): If a non-planarity occurs, then it occurs many times

MAIN THM

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STEPS OF THE PROOF:

1. Non-planar extensions of planar graphs
2. Bounded tree-width
3. Societies with leaps
4. Societies with no large transaction

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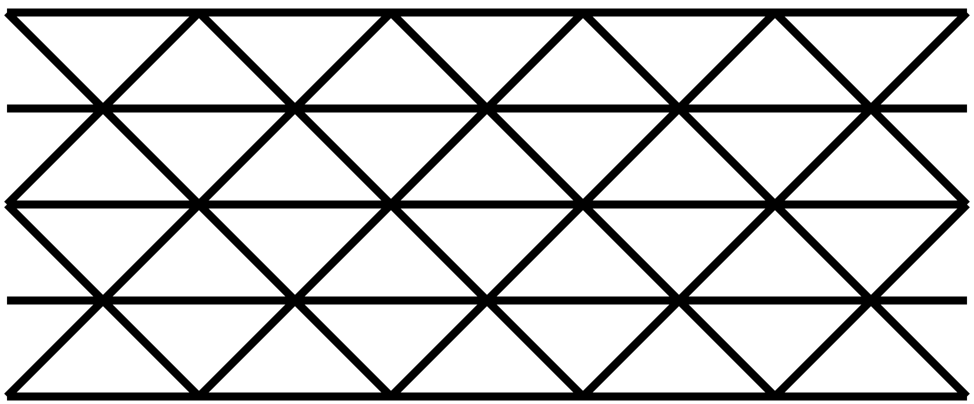

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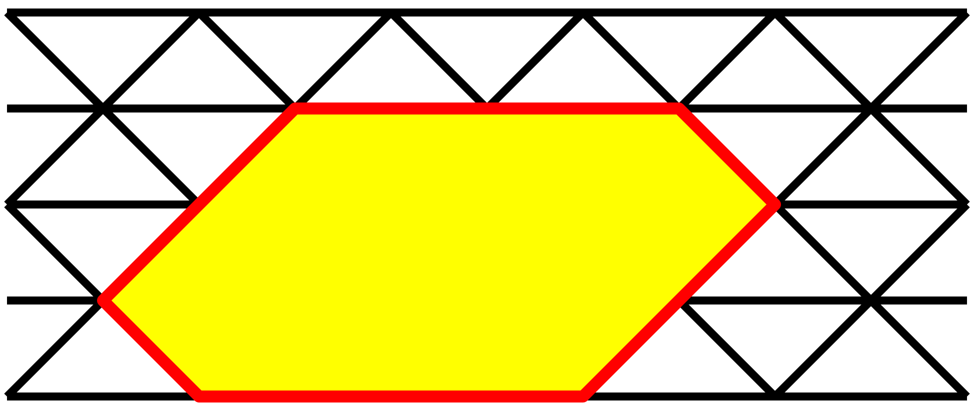
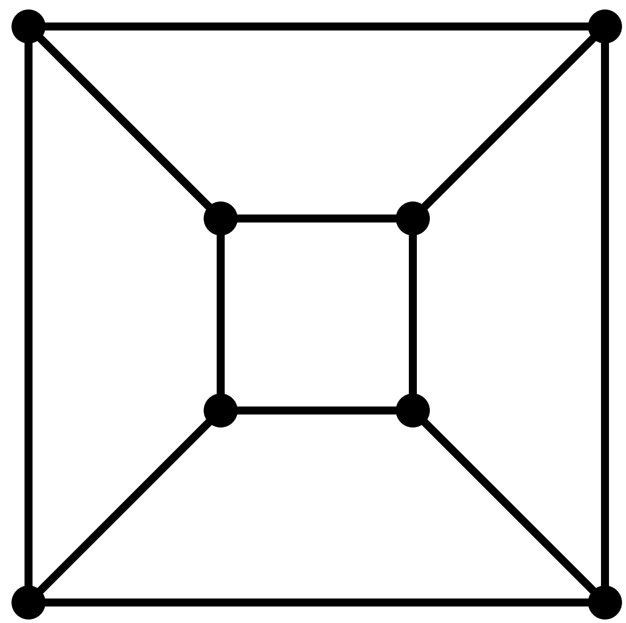


G is internally 4-connected if 3-connected and for every separation of order 3 one side has ≤ 3 edges.



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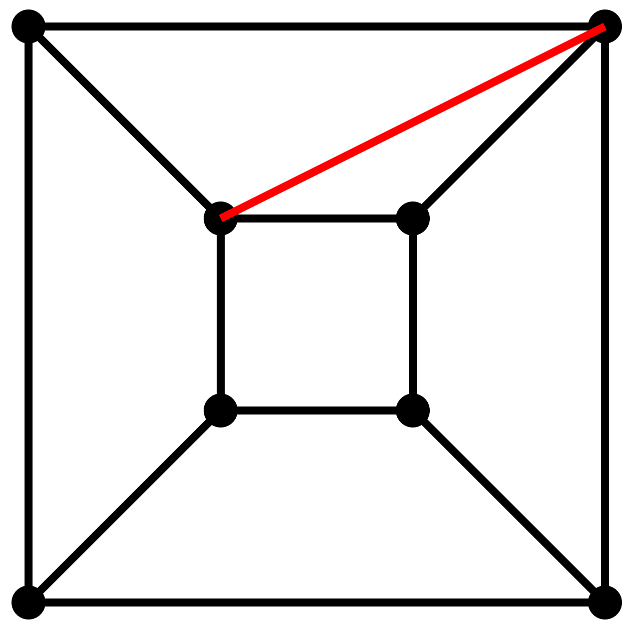

Not i-4-c

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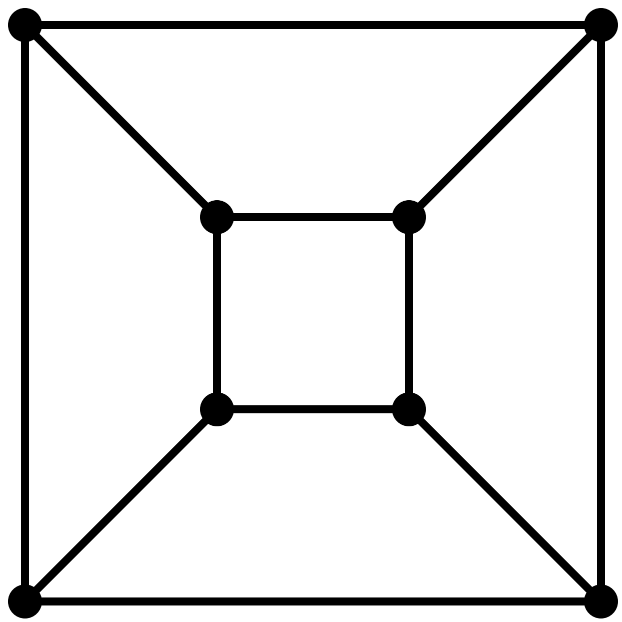

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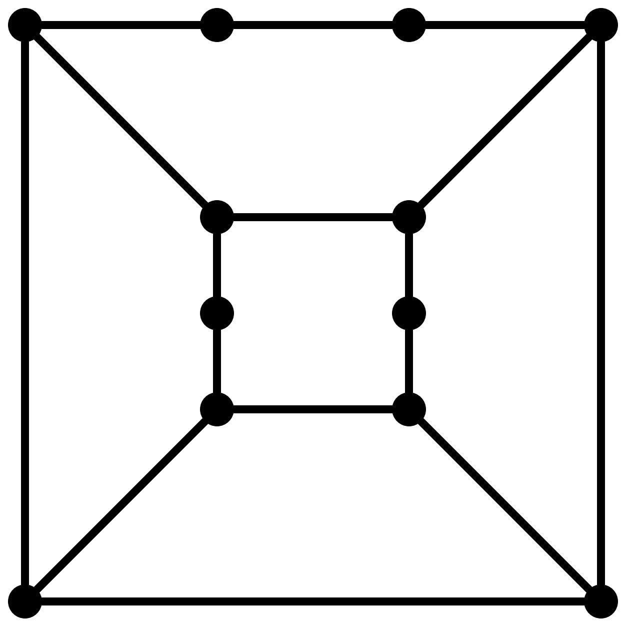

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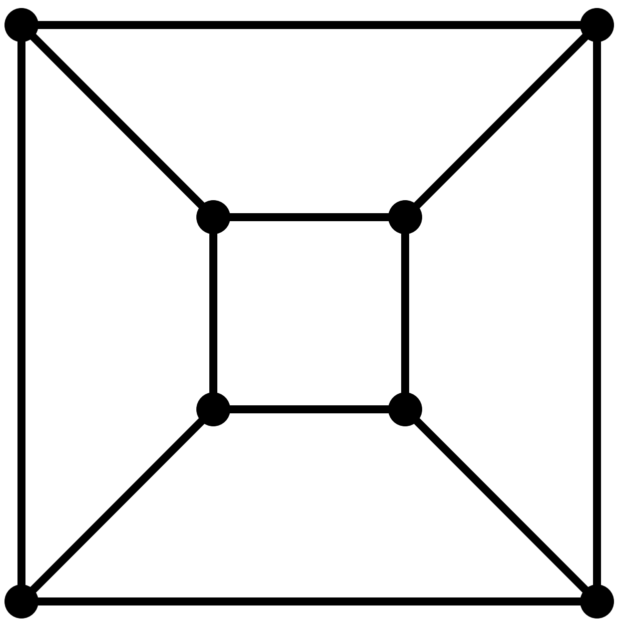
Then G+??$\leq_t H$.



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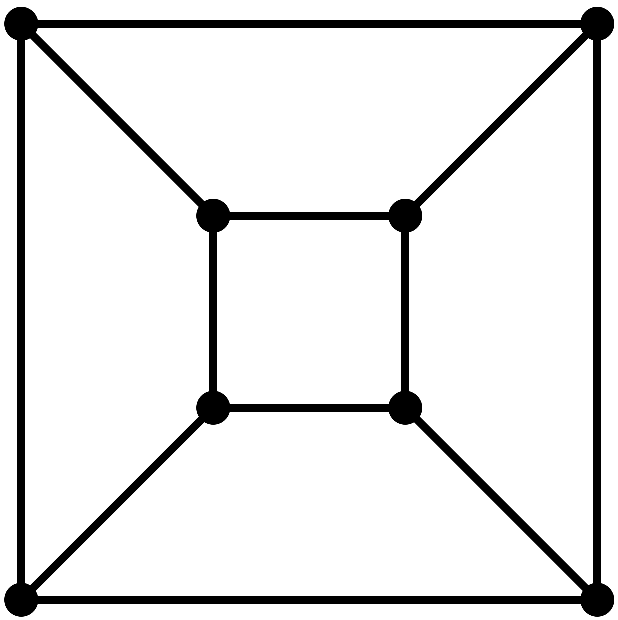

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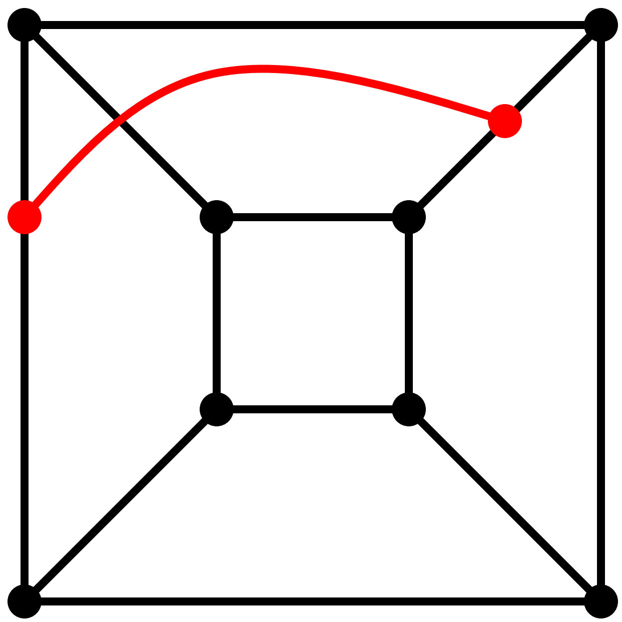

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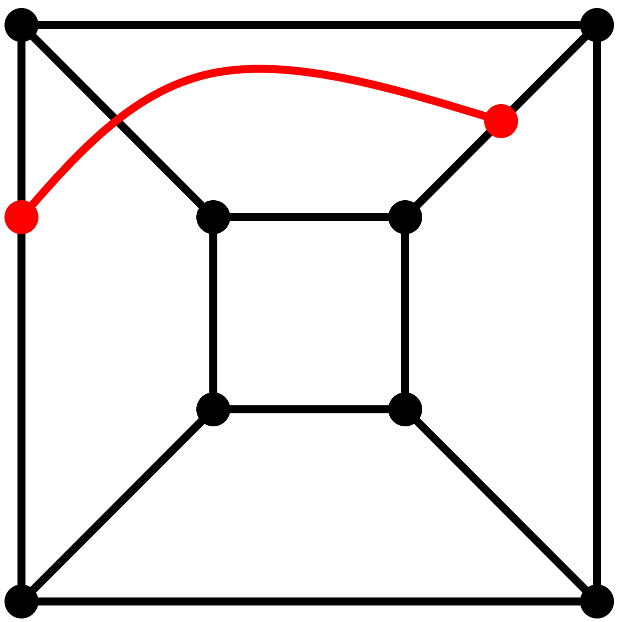

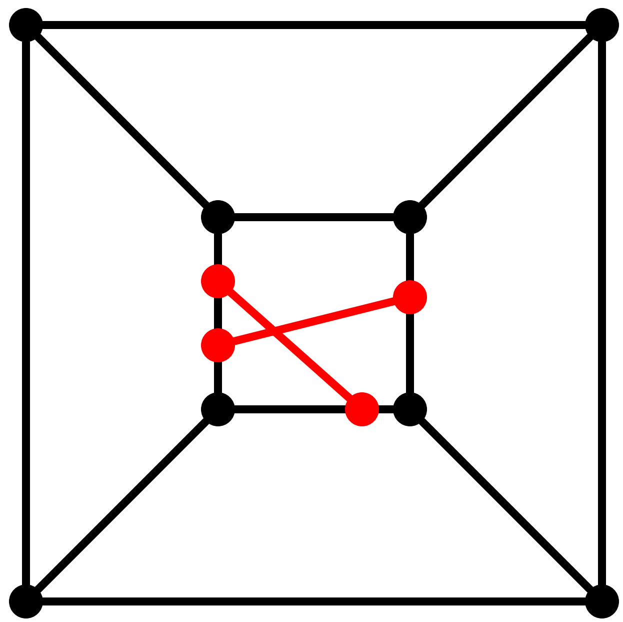

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Extensions

- Rooted graphs
- Graphs on surfaces
- Apex graphs
- Minors

Applications

THM (Ding, Oporowski, Vertigan, RT) Every huge 4-connected non-planar graph has a minor isomorphic to of one of:

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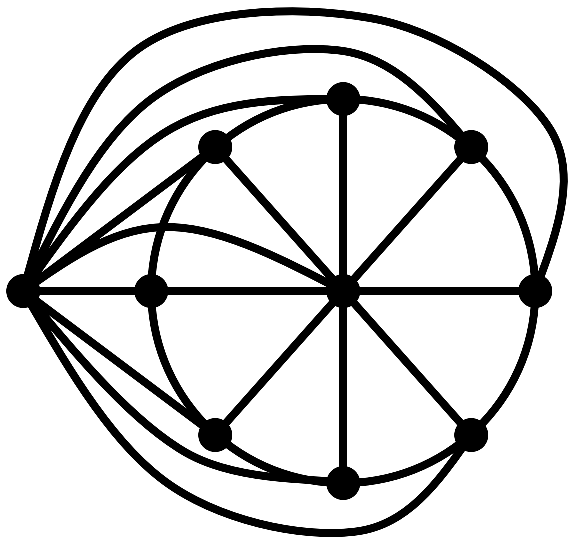

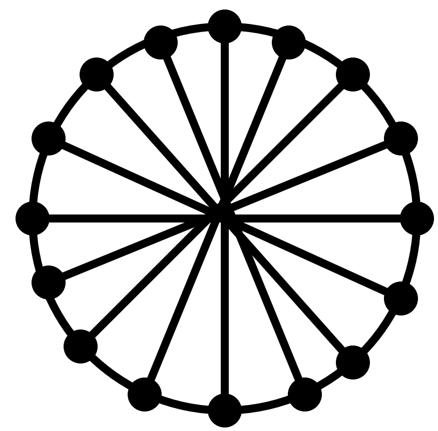

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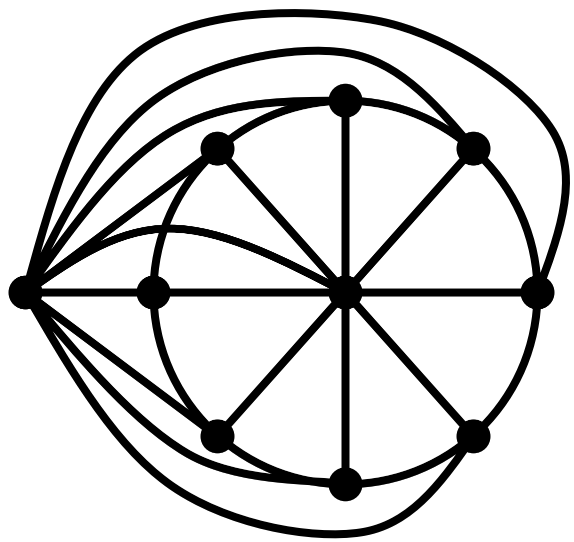

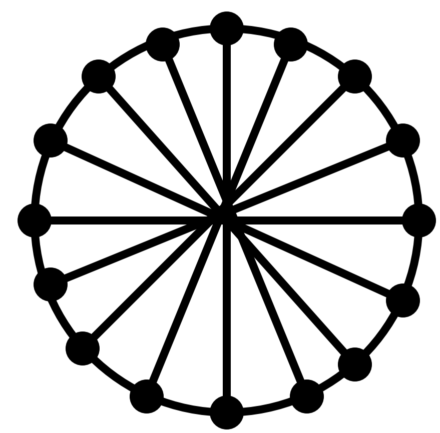

COR Every huge minimal graph of crossing number ≥ 2 contains

Applications

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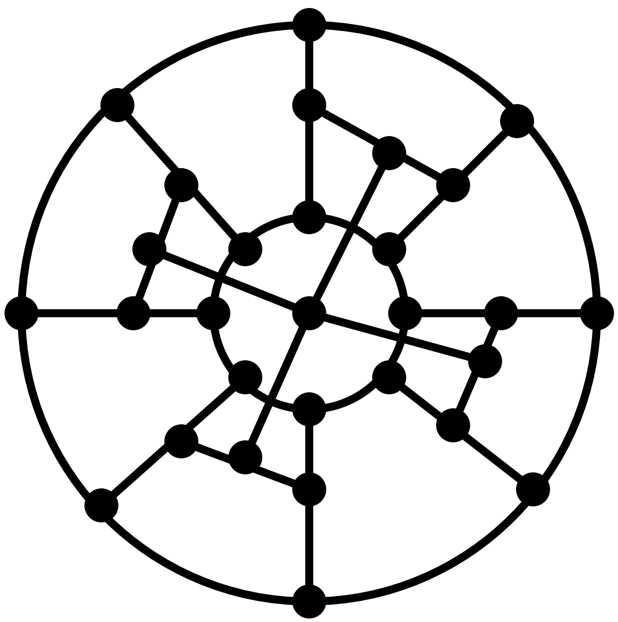
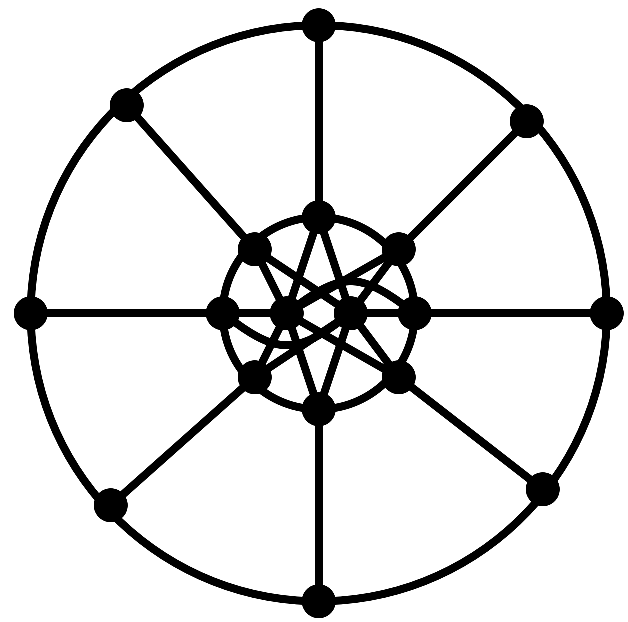
THM A 5-connected graph $G \not\geq_m K_6$ contains a subdivision of a graph below $\Rightarrow$ G is apex.





Bounded tree-width

G has tree-width $<k$ if it has a tree-decomposition into pieces of size $\leq k$.

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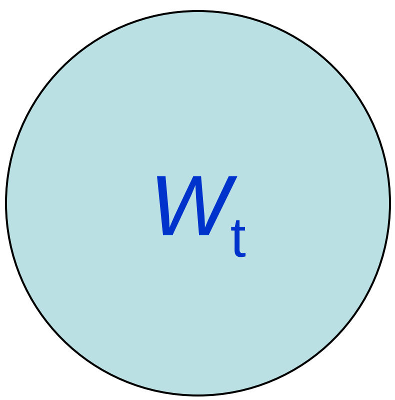

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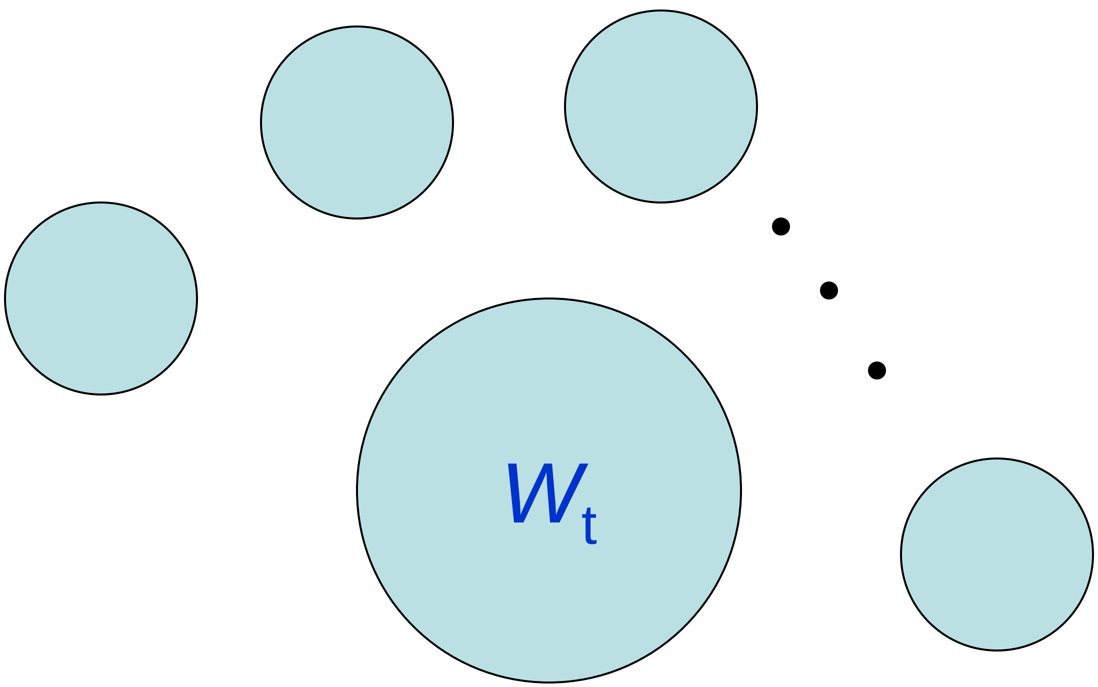

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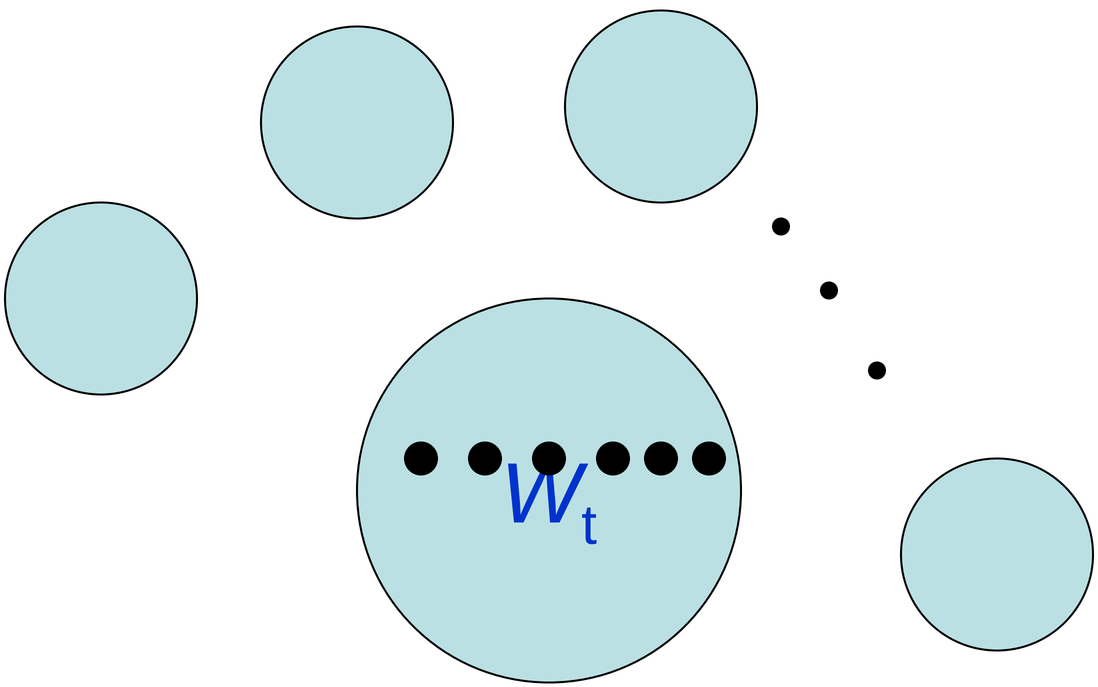

THM $\forall$ k $\exists$ N $\forall$ 6-connected G of tree-width $<k$ on $\geq$N vertices is apex.

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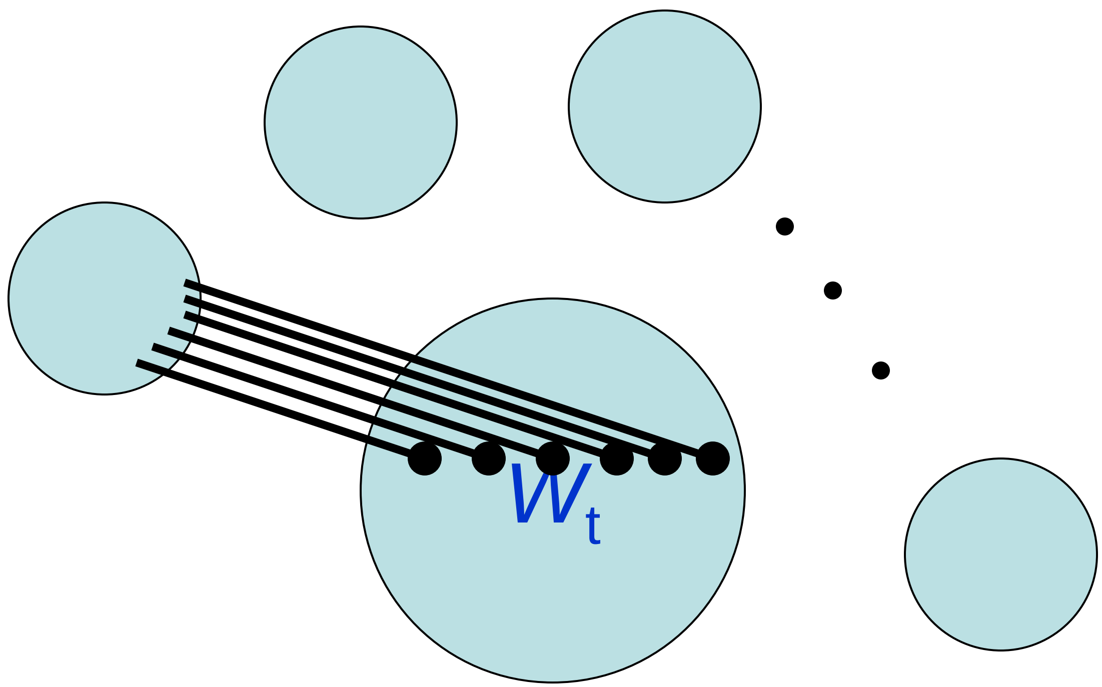

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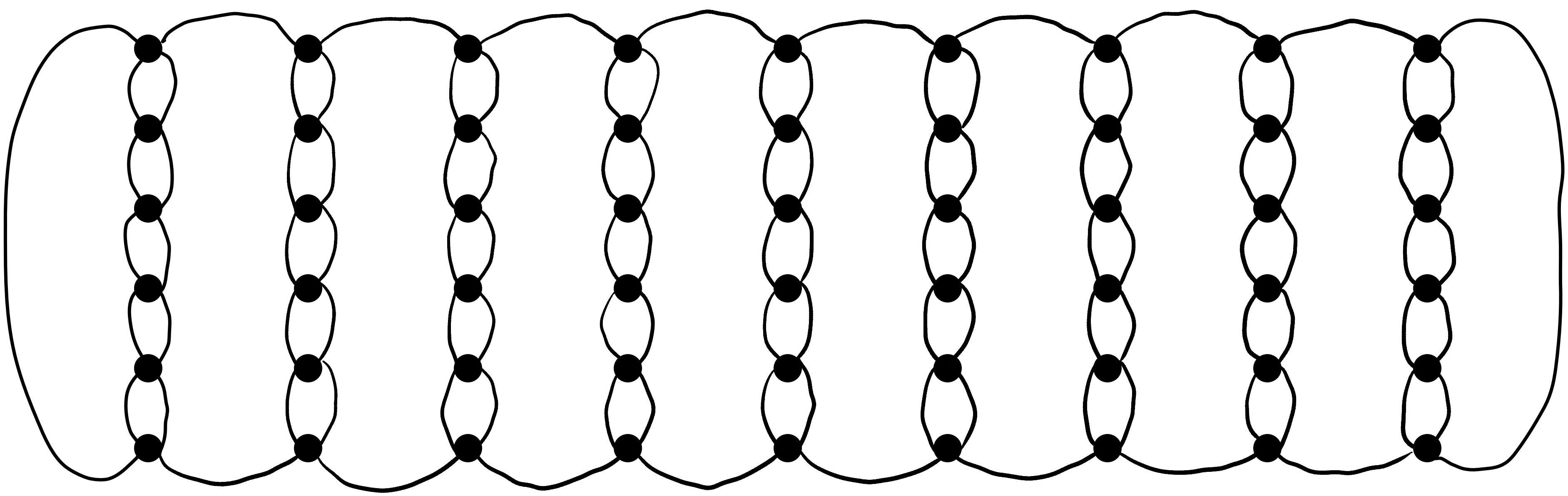

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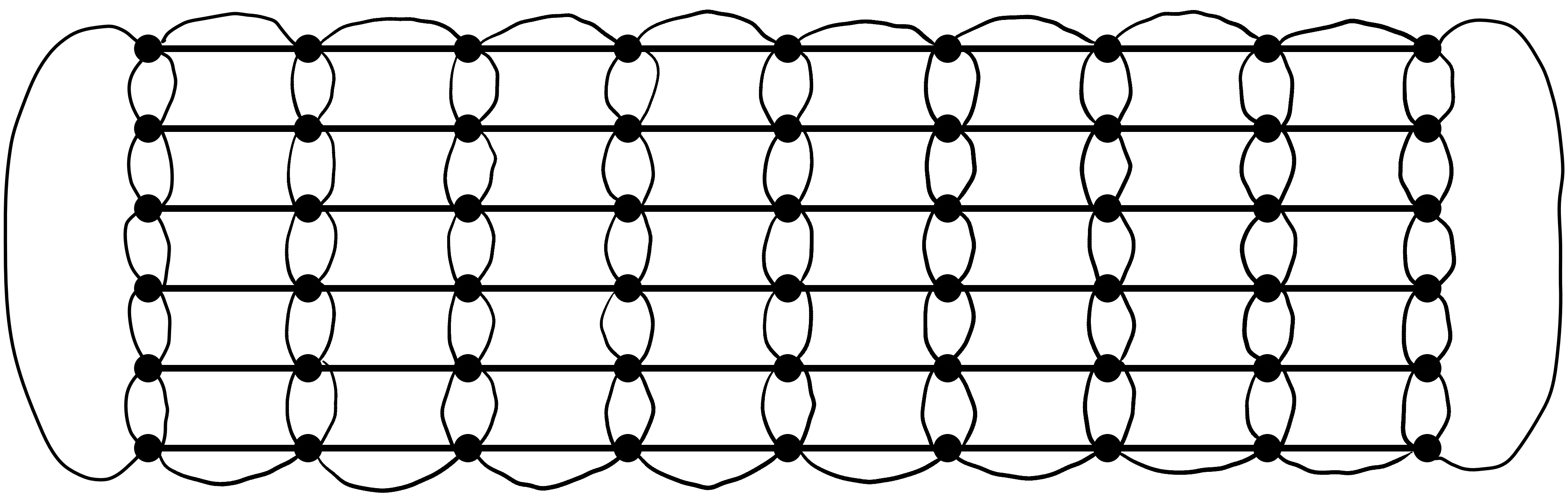

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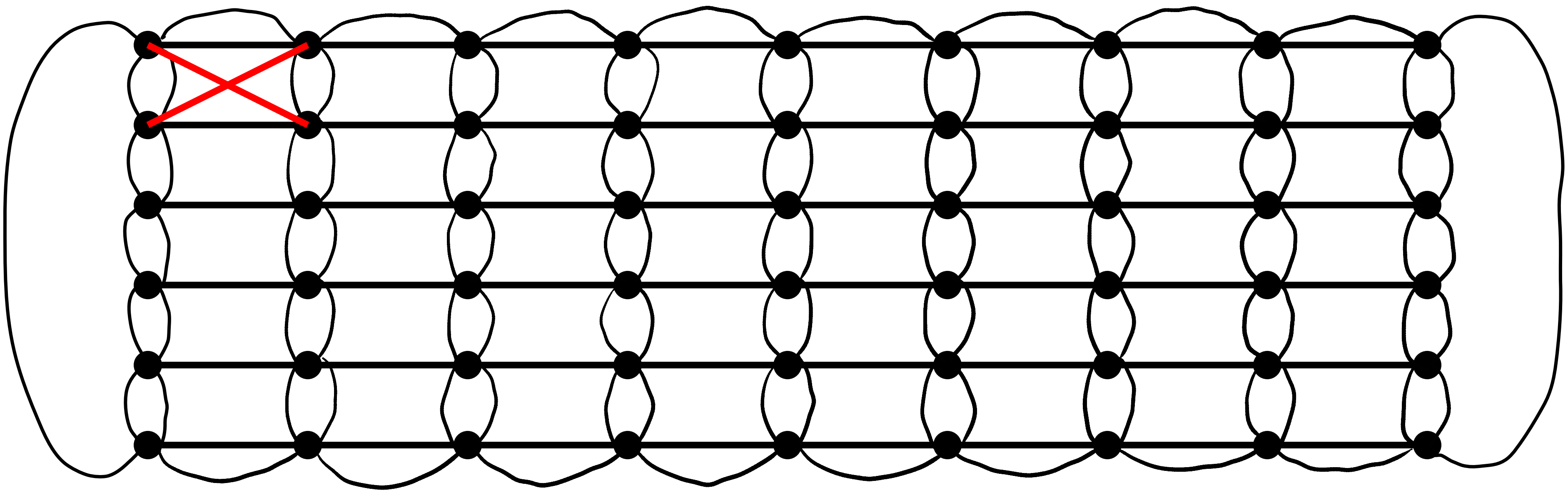

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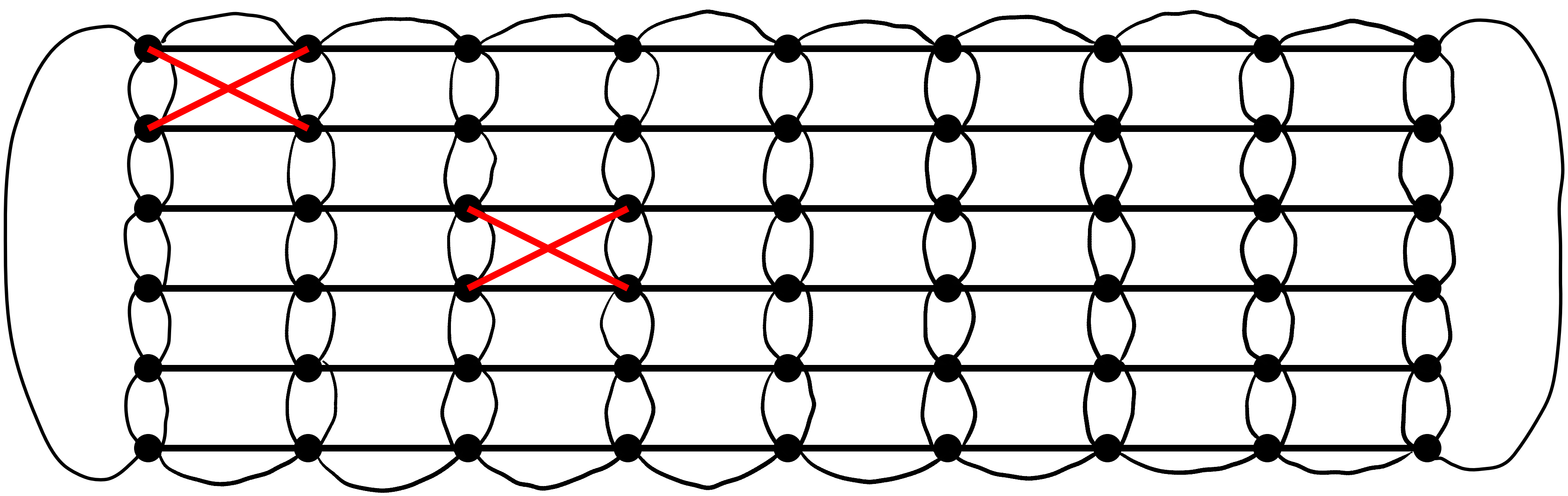

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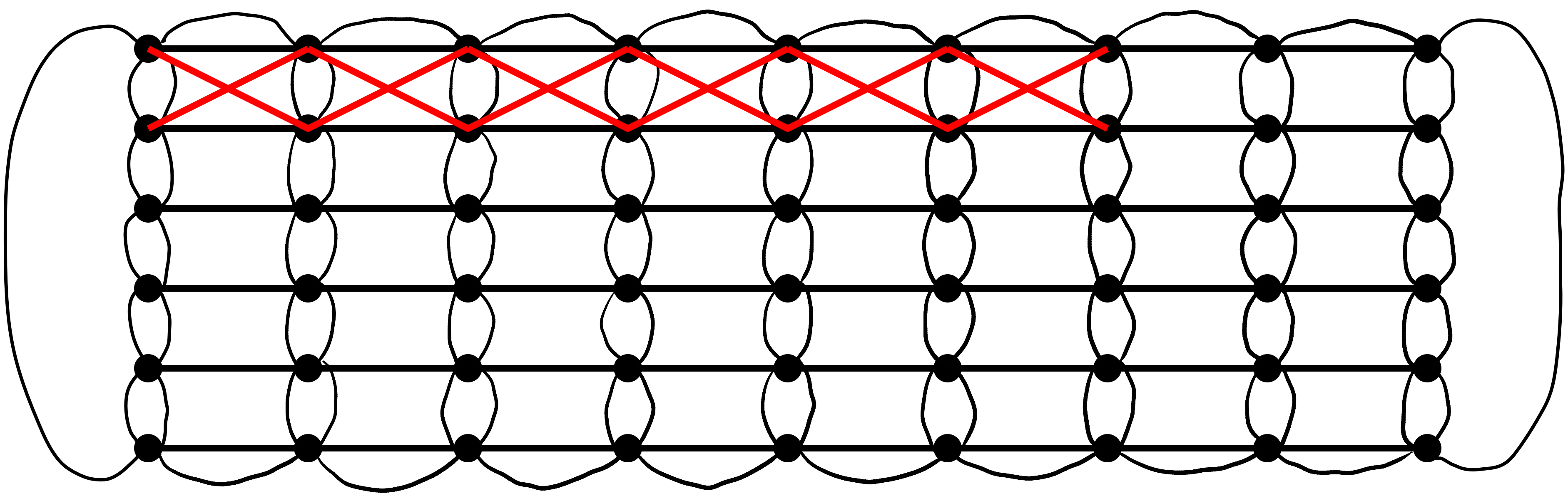

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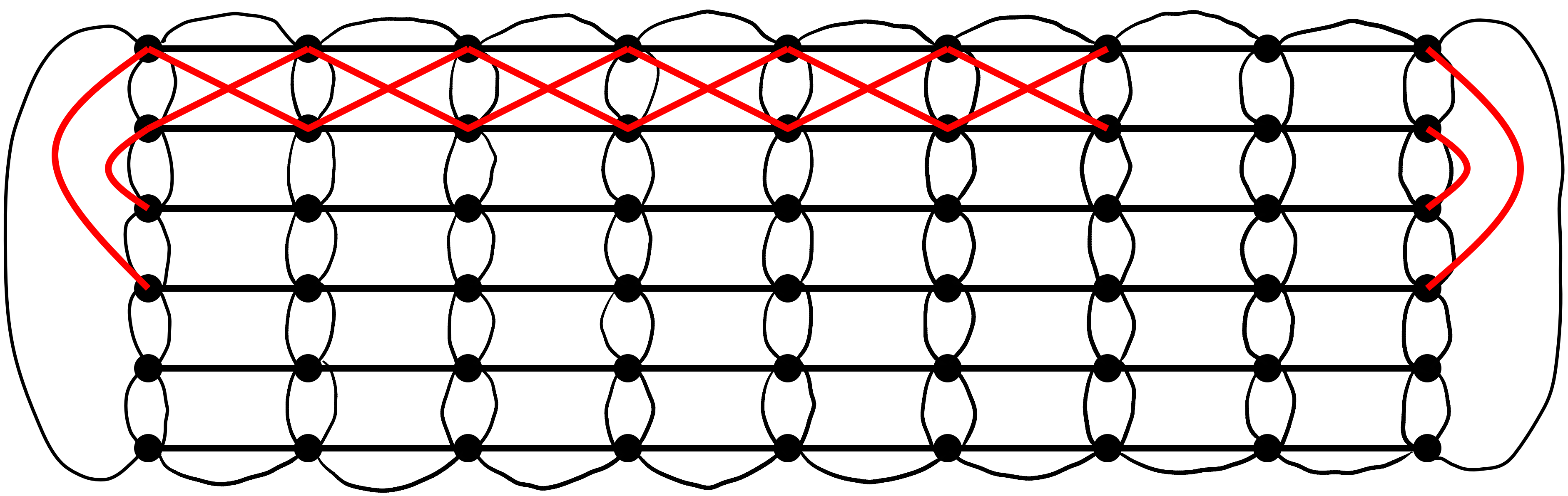
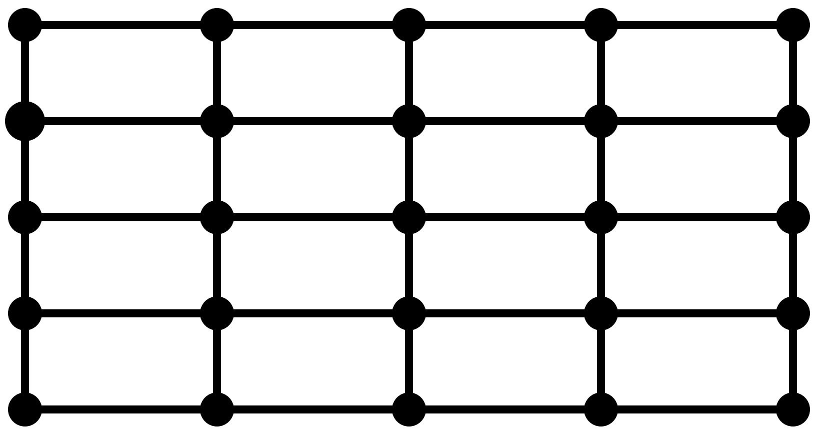


Huge tree-width

THM (Robertson & Seymour) G has huge t.w.
$\Rightarrow$ G has a large grid minor



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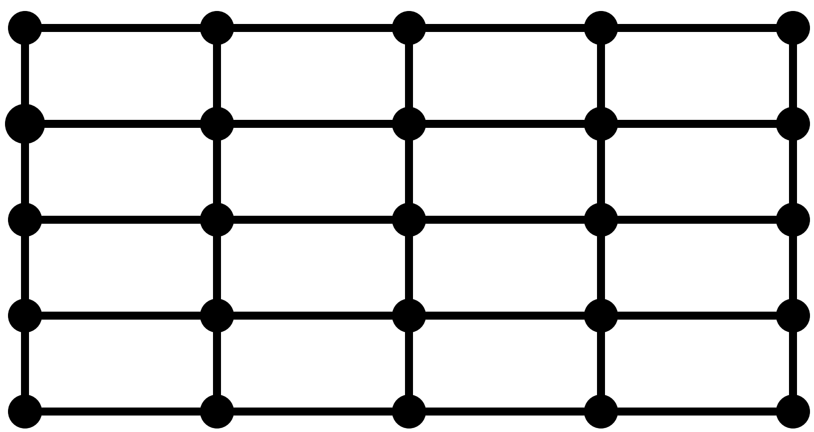

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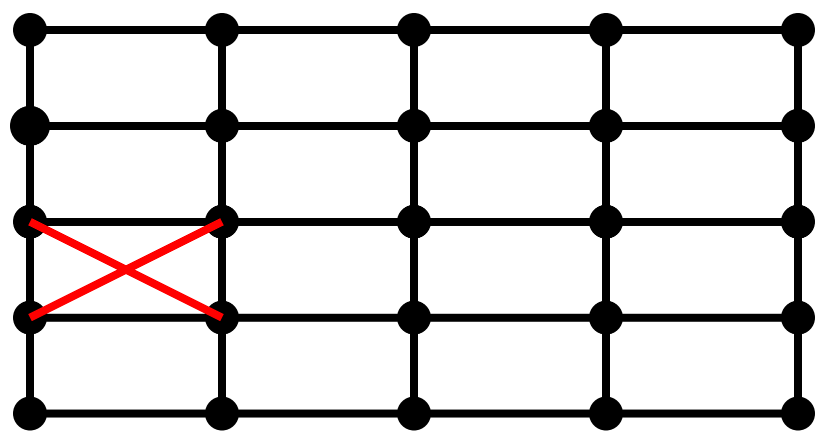

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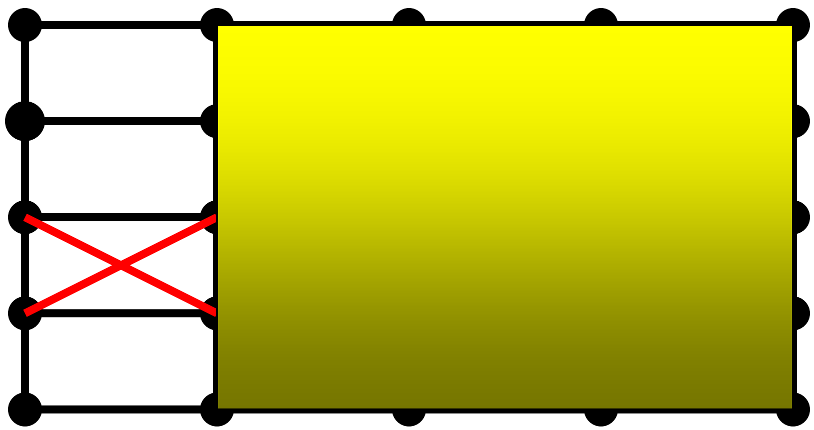

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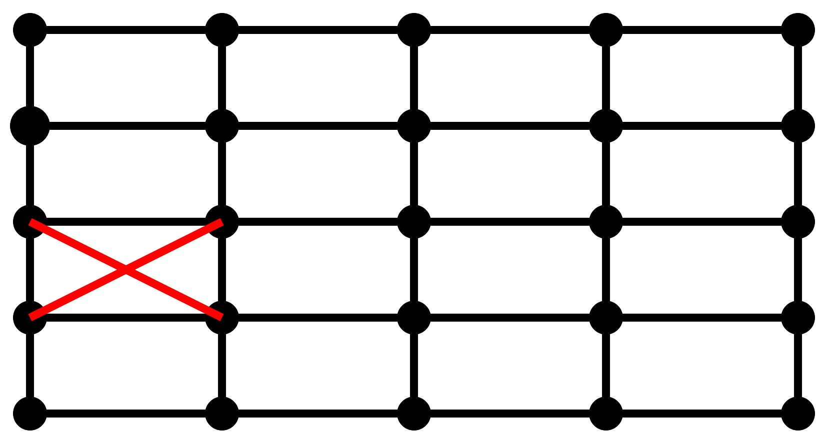

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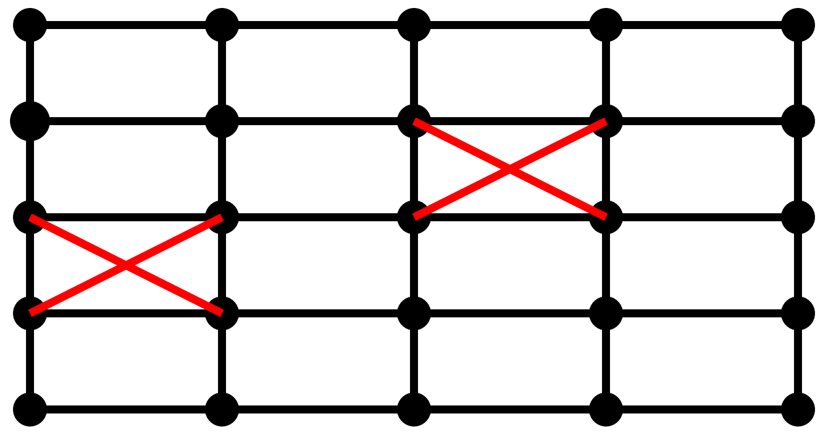

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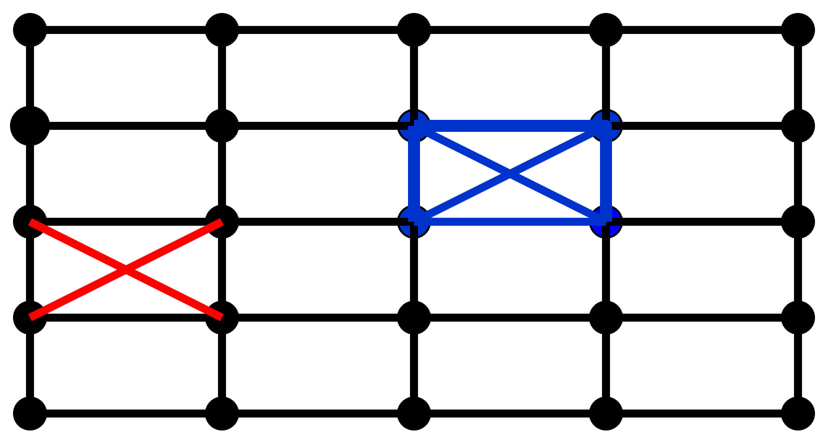

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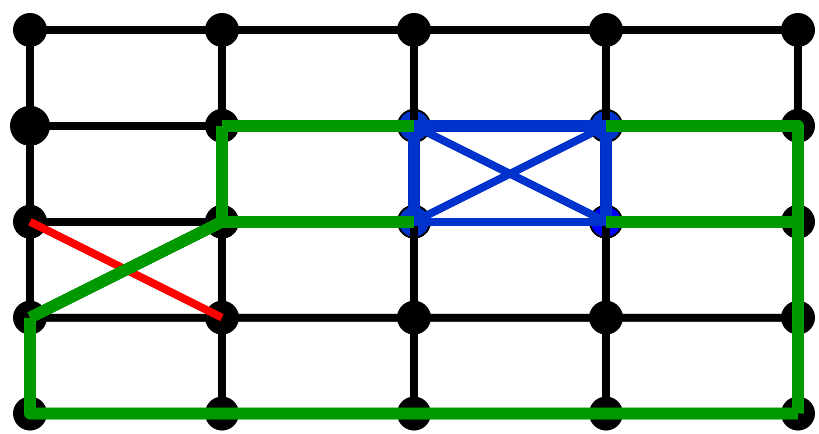

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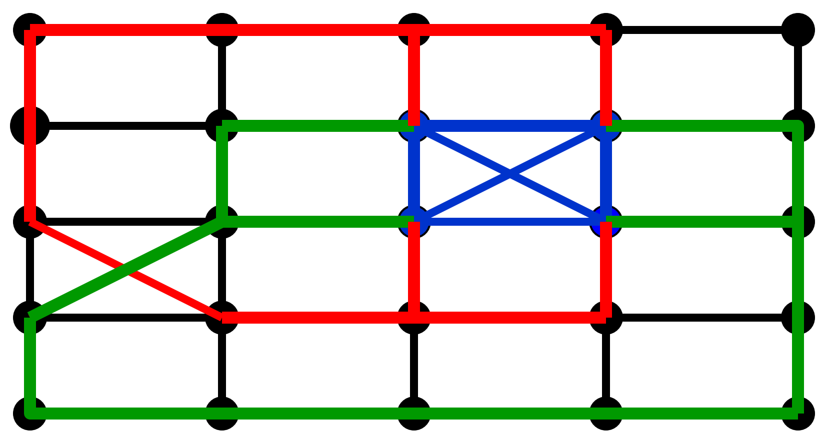

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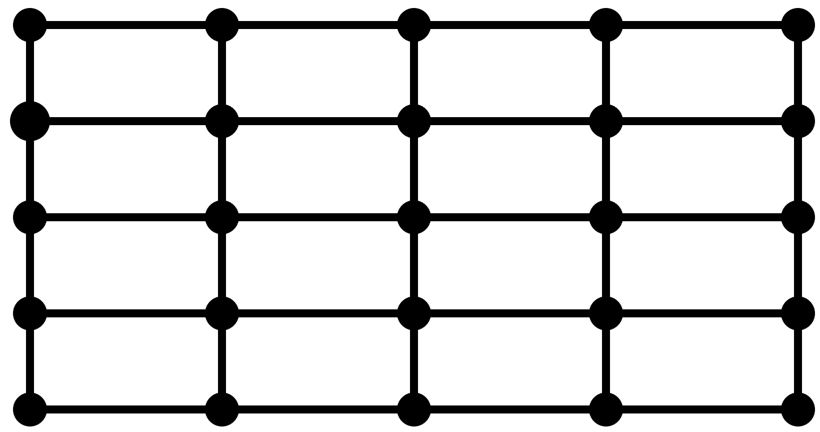

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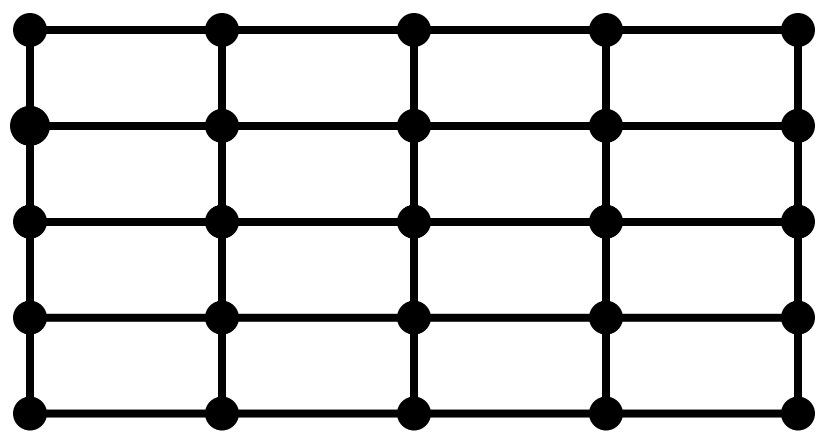

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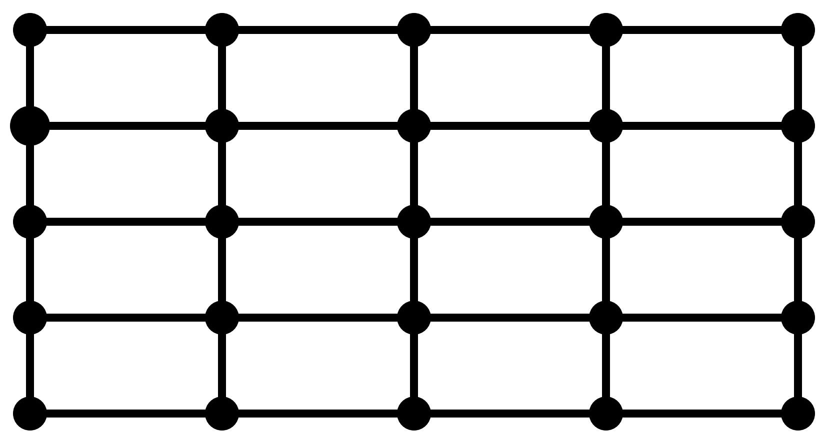

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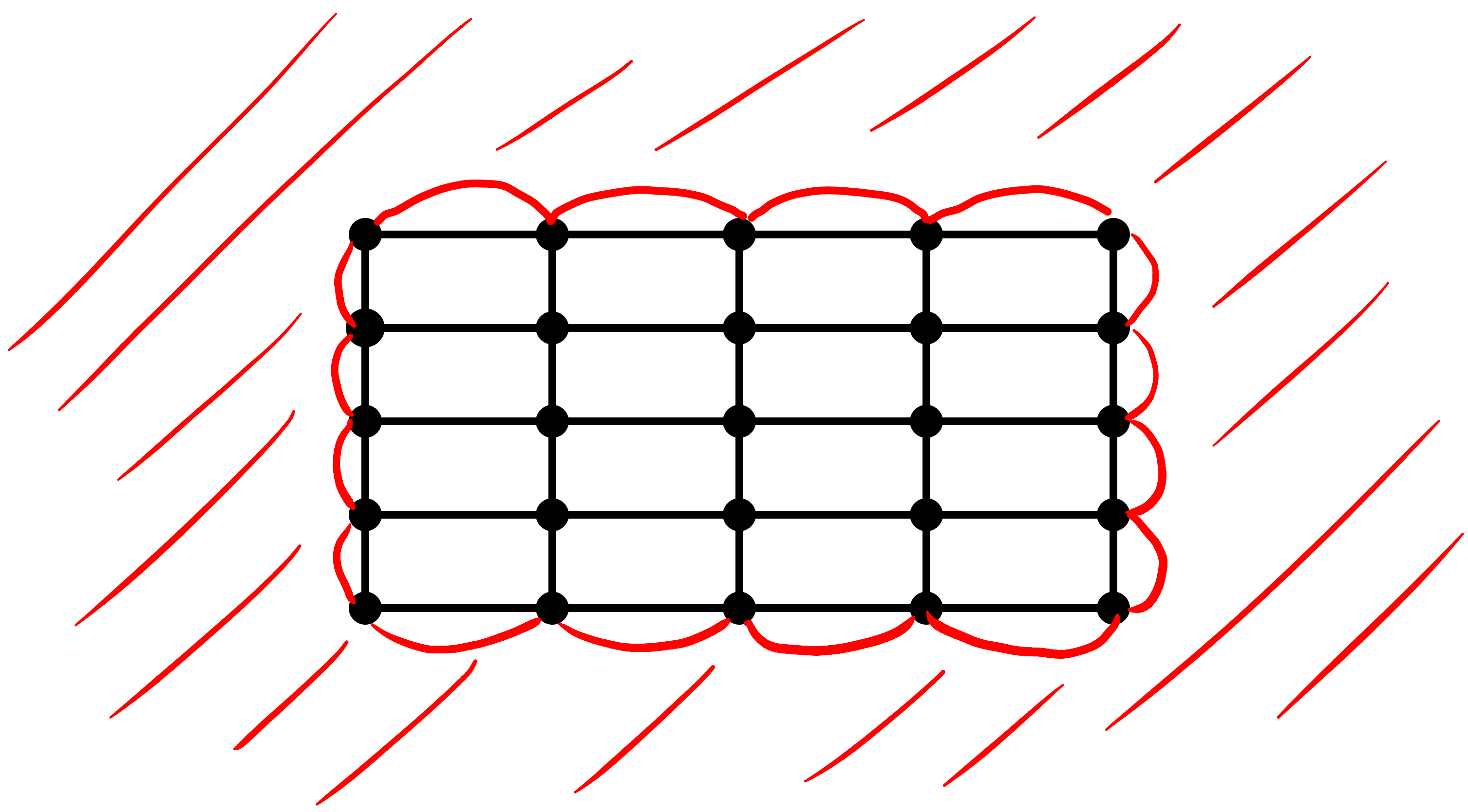
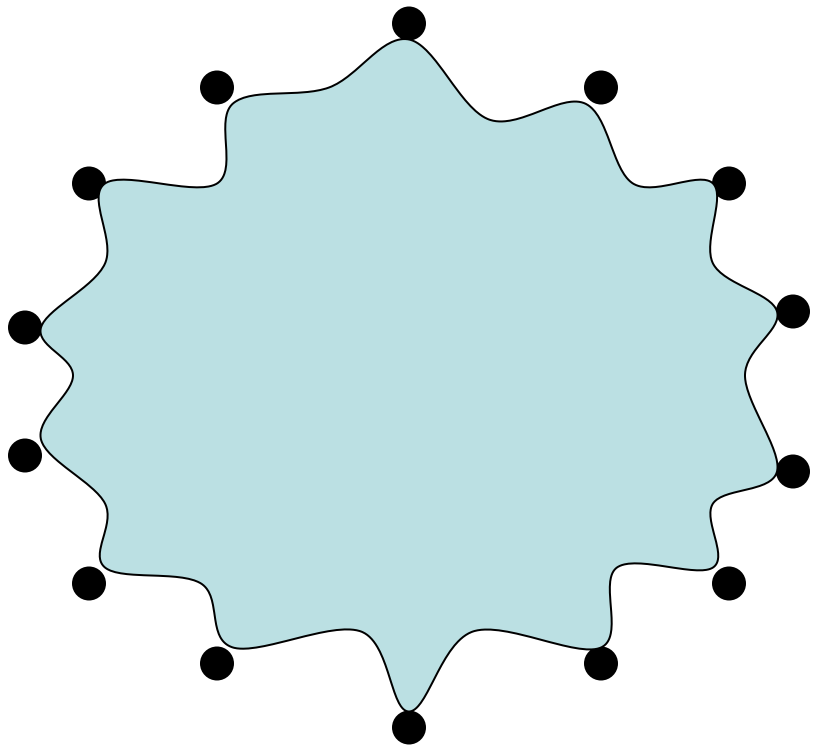
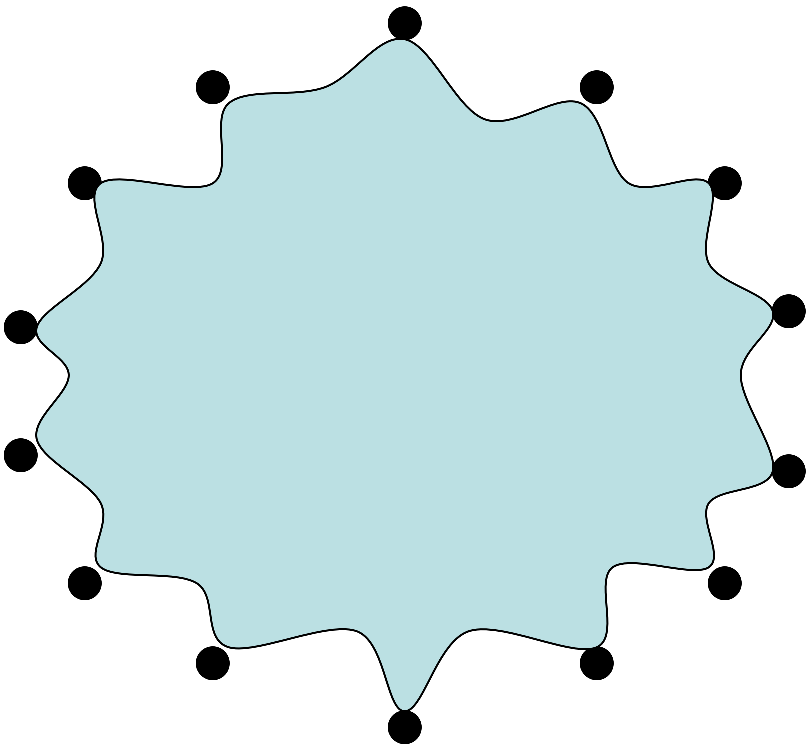
Let's look at the outside of the grid minor



DEF A society is a pair (G,Ω), where G is a graph and Ω is a cyclic ordering of a subset $V(\Omega) \subseteq V(G)$.



DEF A transaction in a society



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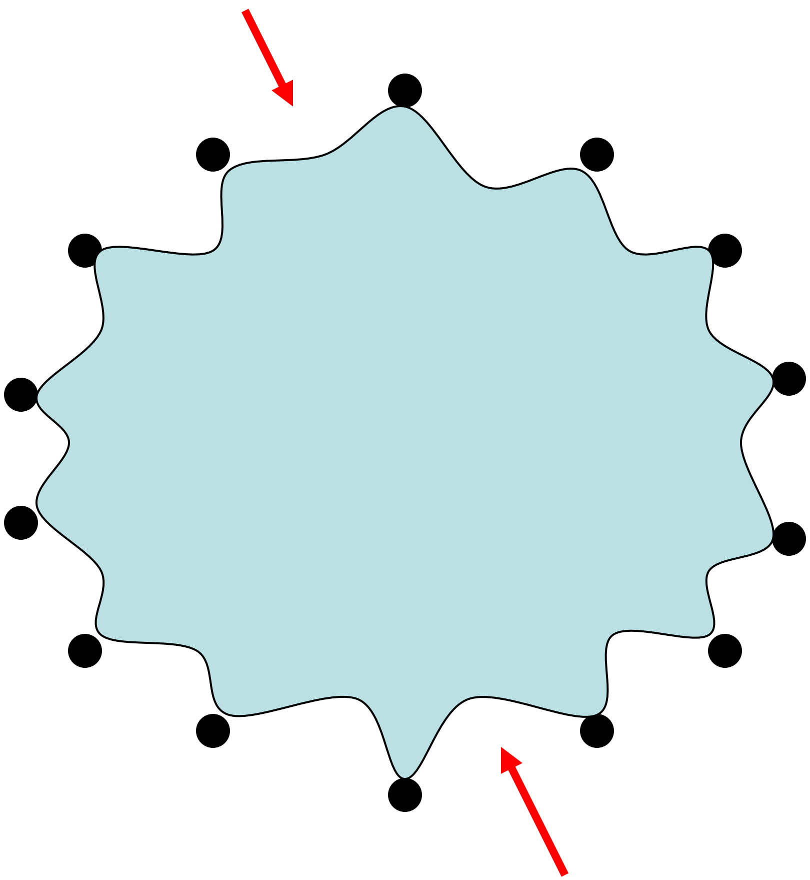

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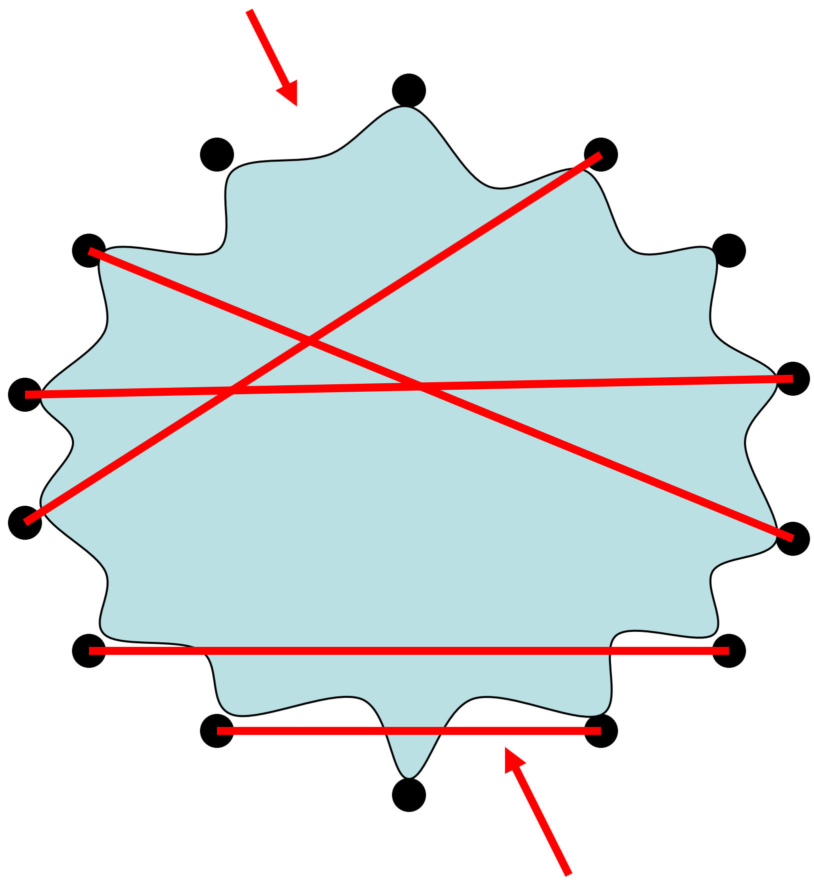
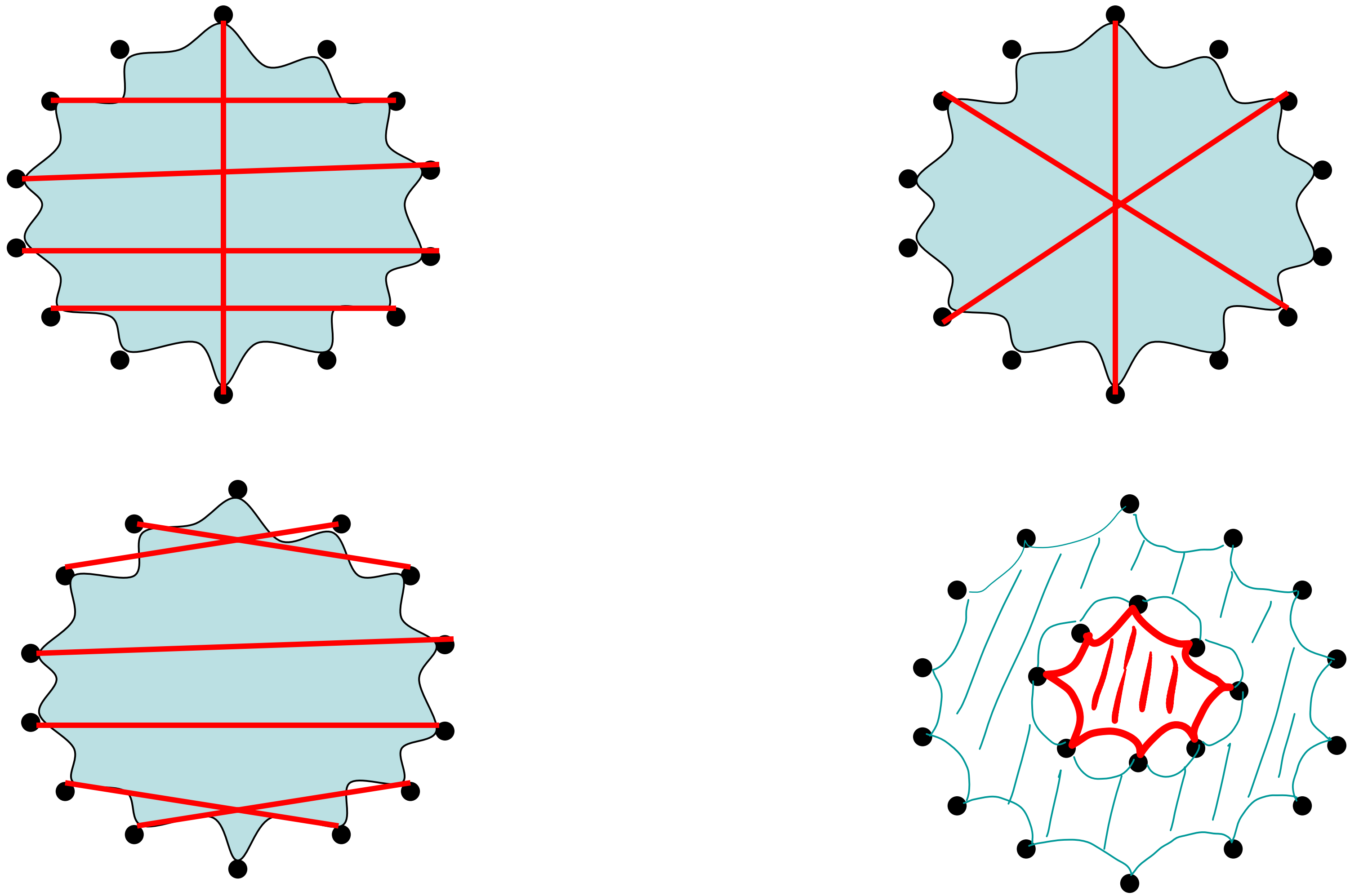


THM (Robertson & Seymour, Graph Minors IX)

Every society satisfies one of the following:



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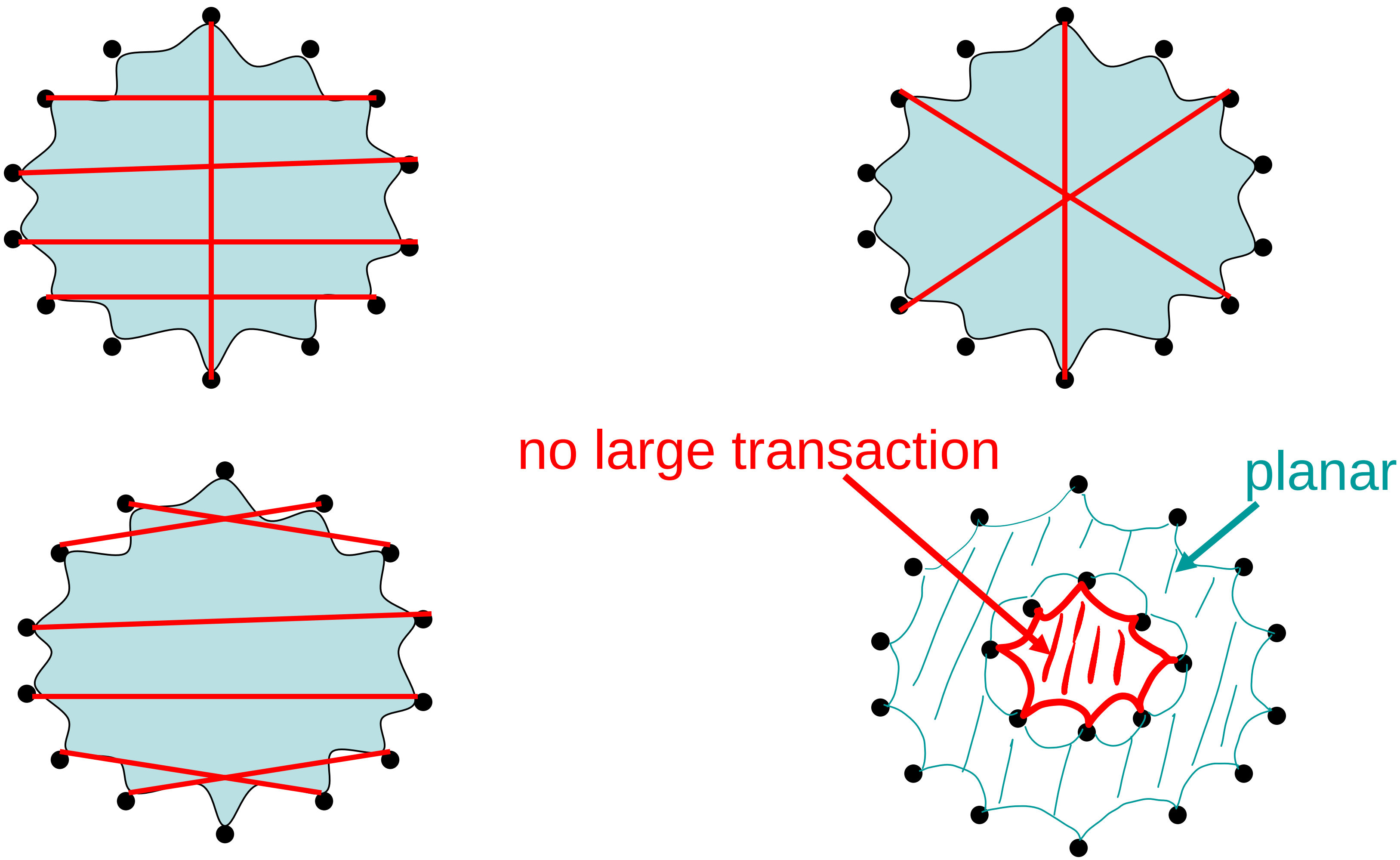
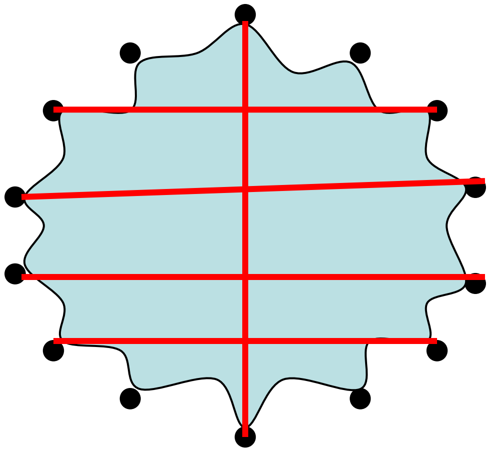
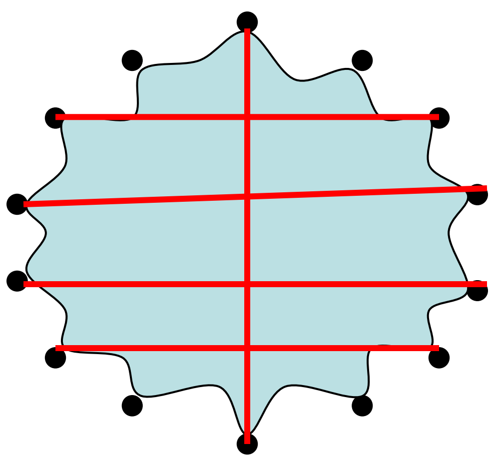




THM For a society (G,Ω) with a 4-leap, the following are equivalent:



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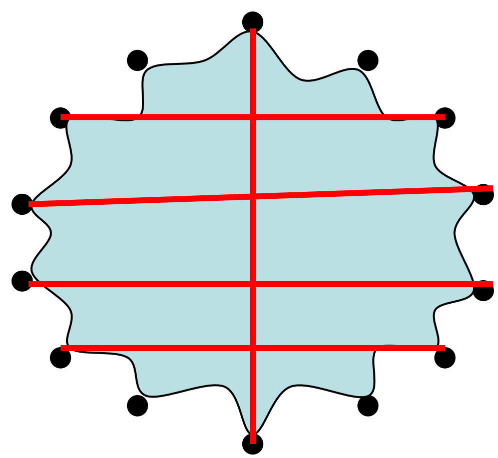

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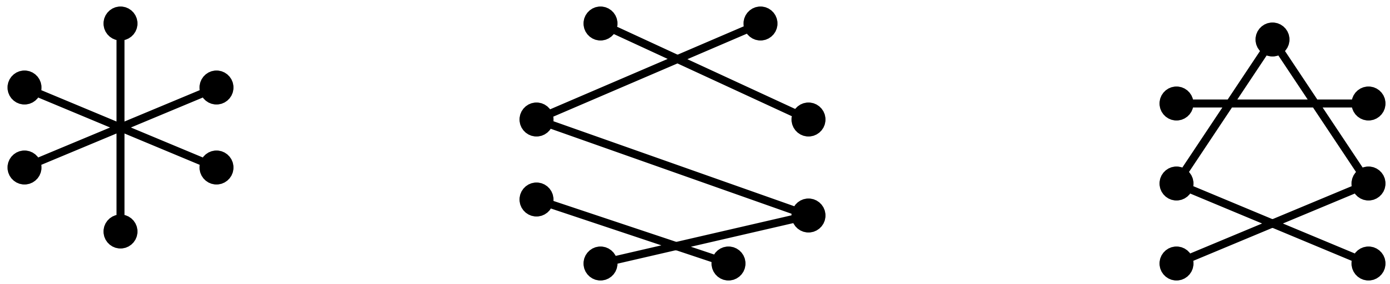

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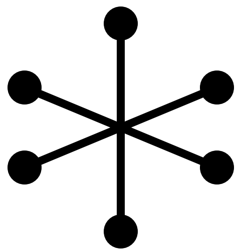
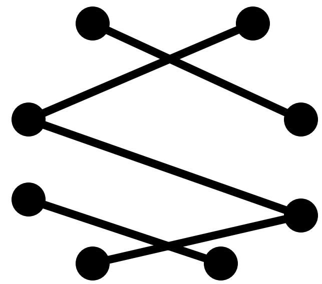
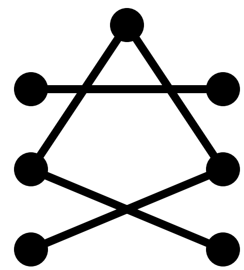
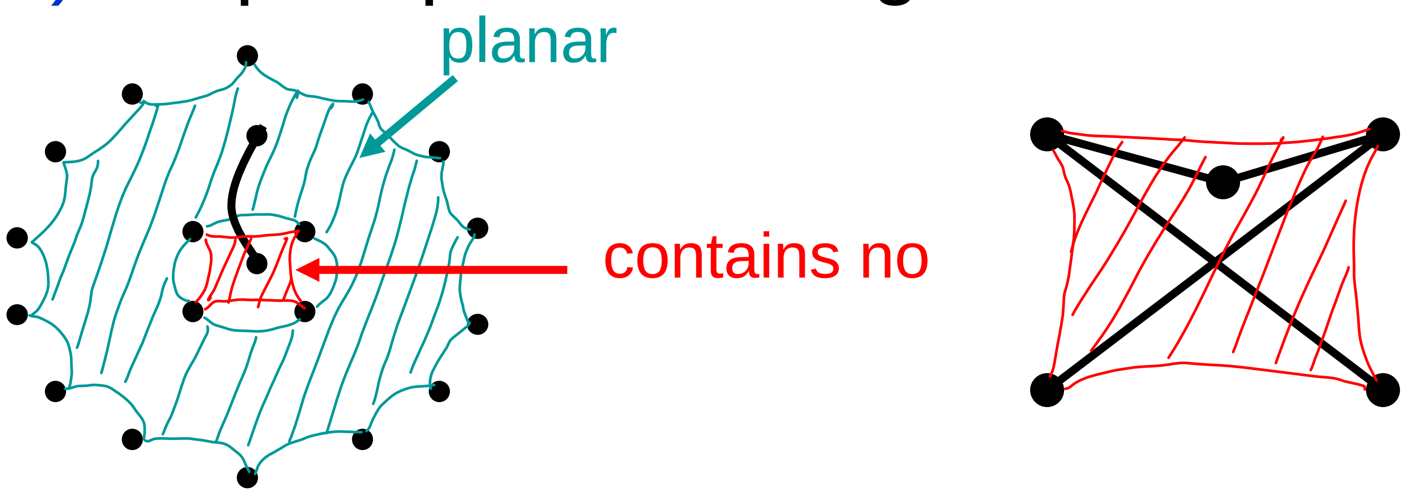






(3) (G,Ω) is apex, planar+triangle or



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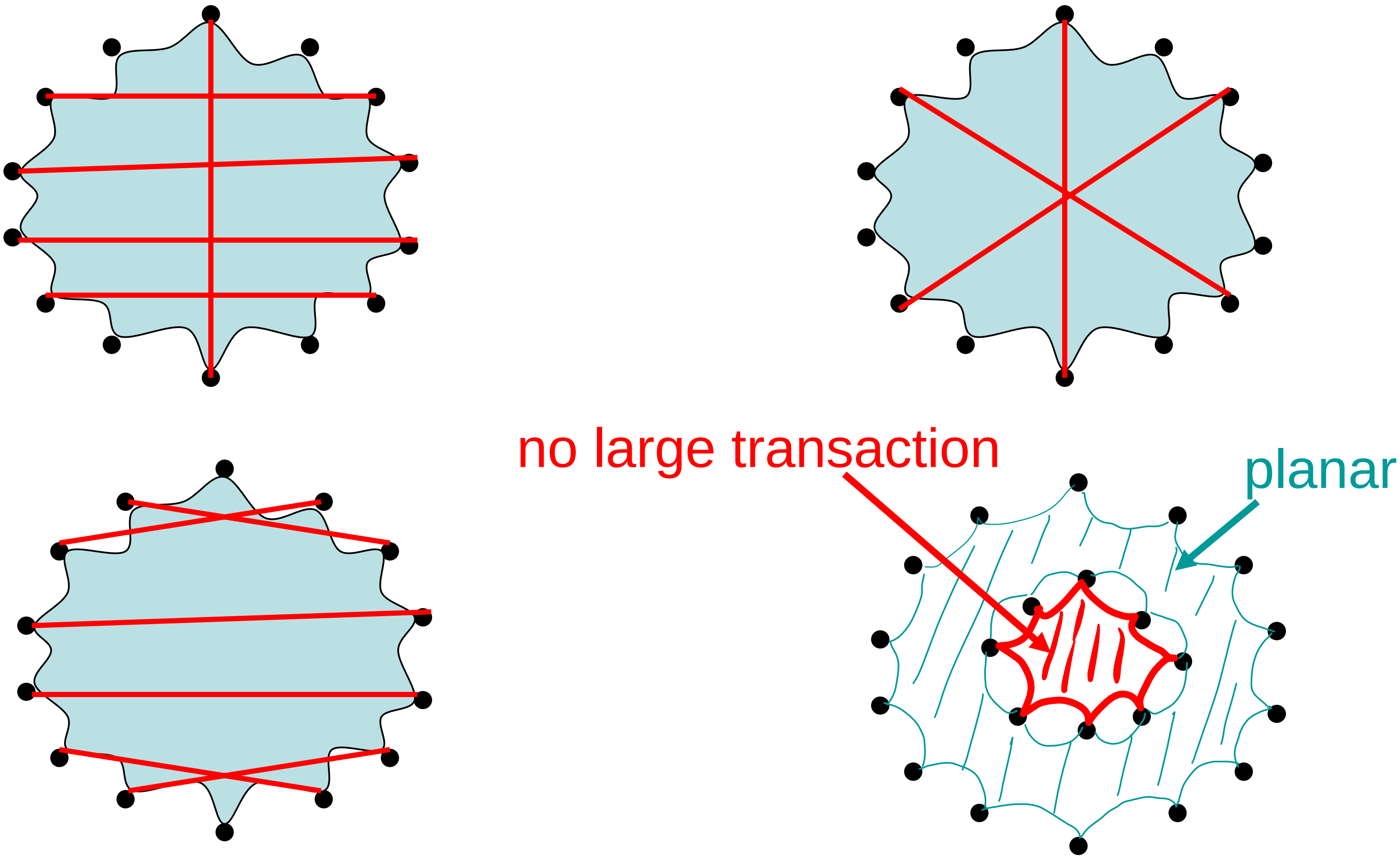
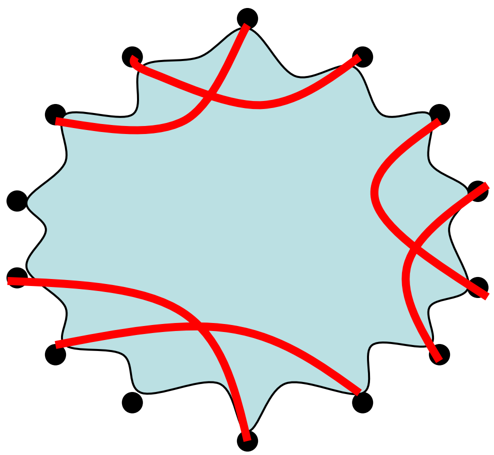
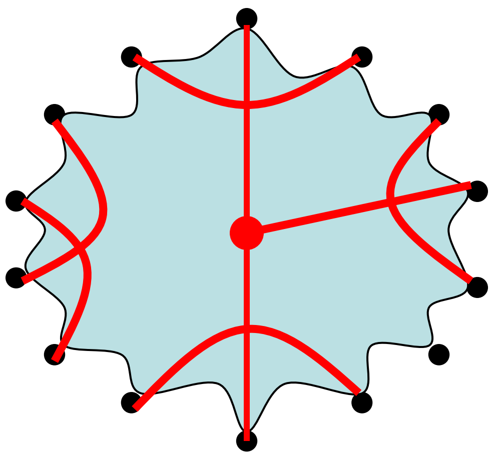
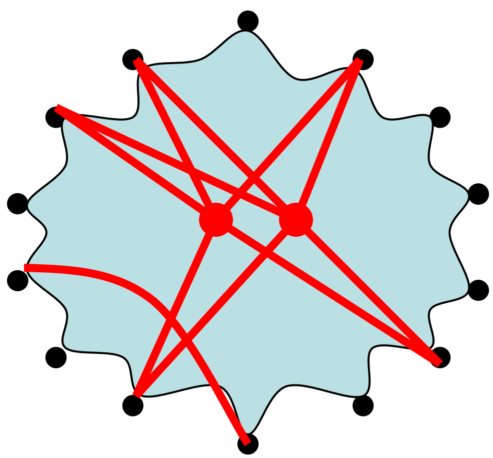


THM Every internally 6-connected non-apex society with “many legs” and no large transaction contains:







plus 2 small variations

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CONJ (Seymour, RT) G is $(t$-$2)$-connected, big $G \not\geq K_t \Rightarrow |E(G)| \leq (t-2)n-(t-1)(t-2)/2$