PERFECT GRAPHS

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Georgia Institute of Technology
and
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joint work with

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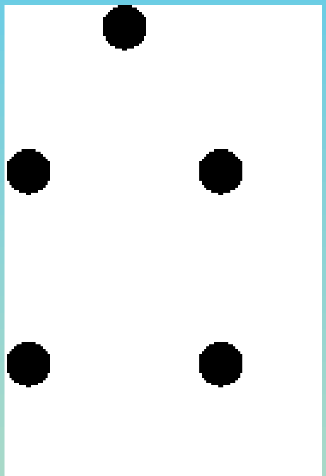
PART I

History and relevance of perfect graphs

PART II

The strong perfect graph conjecture

Graphs have vertices



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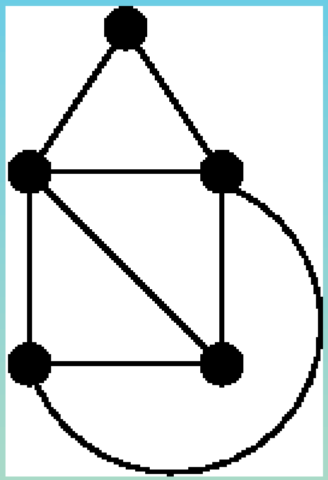

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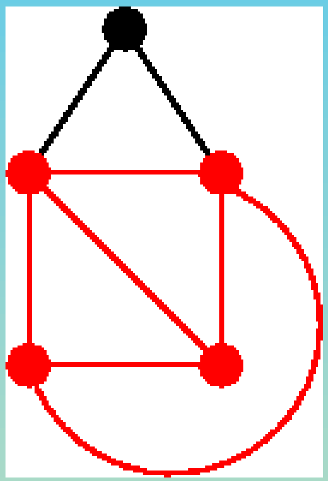

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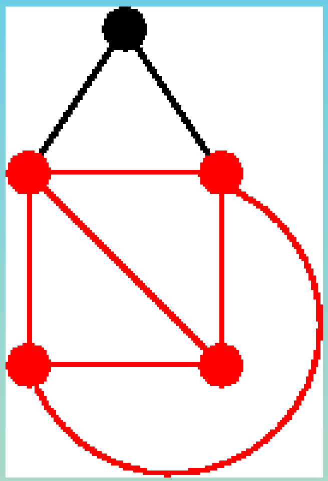

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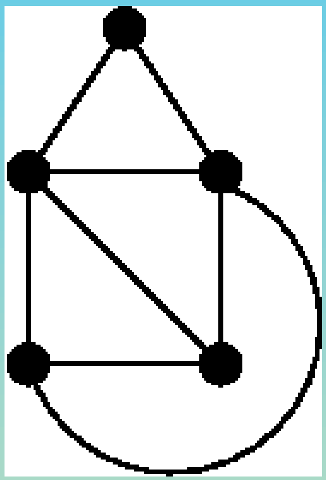

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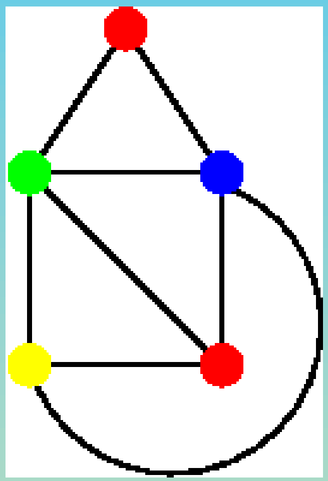

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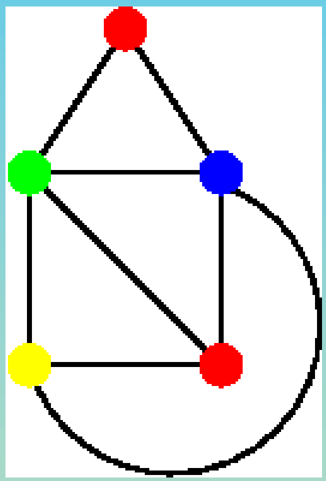

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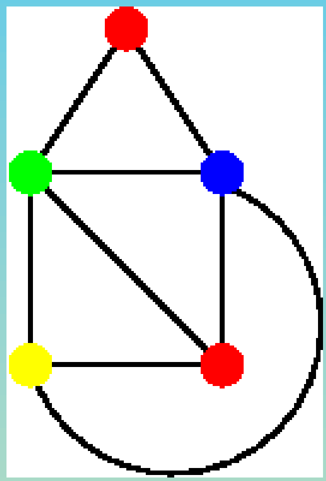

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Graphs that have an odd hole or odd antihole

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Bipartite graphs

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THE STRONG PERFECT GRAPH CONJECTURE (SPGC) (Berge 1960)
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But ab, bd, ca, dc, ee are pairwise unconfoundable
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- Fundamental and beautiful open problems

THEOREM (Lovász) Let A be a $0,1$-matrix. For every non-negative objective function c the LP

$$\text{max } c^T x \text{ subject to } x \geq 0 \text{ and } Ax \leq 1$$

has integral optimum $\Leftrightarrow$ the undominated rows of A form the vertex versus maximal cliques incidence matrix of some perfect graph.

PART II

The Strong Perfect Graph Conjecture

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MAIN THEOREM Every Berge graph is either basic, or has a certain decomposition.

THE SPGC WAS KNOWN FOR

- planar graphs (Tucker)
- claw-free graphs (Parthasarathy, Ravindra)
- K_4-free graphs (Tucker)
- diamond-free graphs (Tucker)
- bull-free graphs (Chvátal, Sbihi)
- dart-free graphs (Sun)
- C_4-free graphs (Conforti, Cornuéjols, Vušković)
- "wheel-and-parachute-free" graphs (Conforti, Cornuéjols)

NOTE All of the above exclude specific graphs.

To prove the SPGC we must show that every Berge graph G satisfies $\chi(G) = \omega(G)$.

REMINDER Berge means no odd hole or antihole

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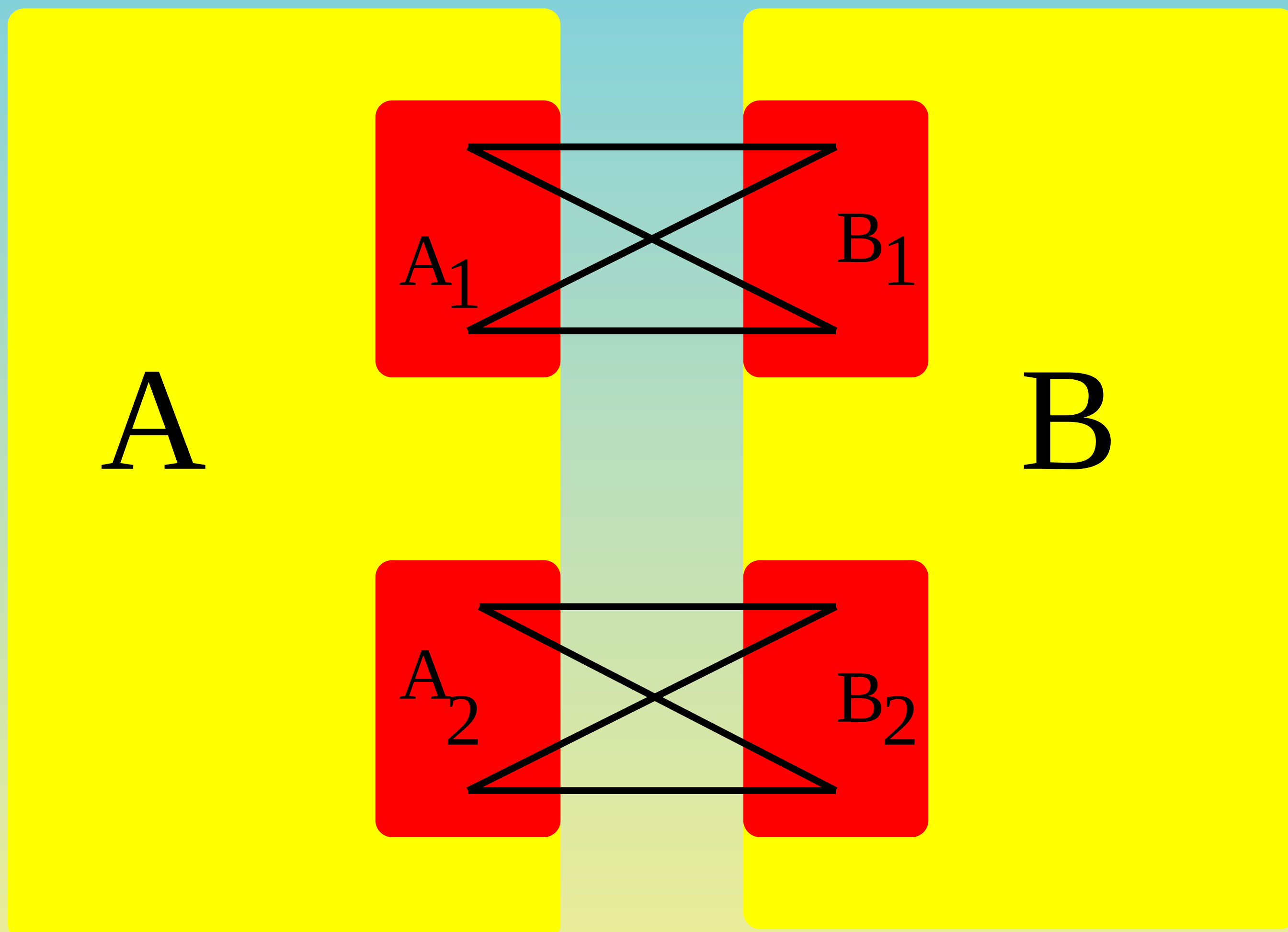

STAR CUTSETS

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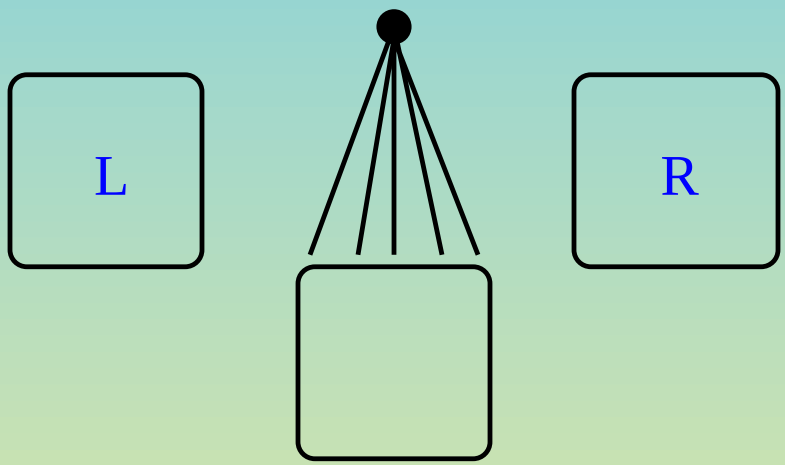

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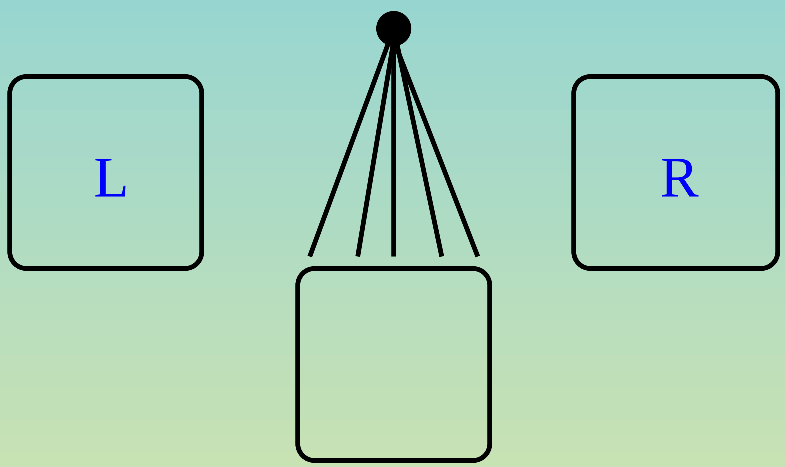

THEOREM (Chvátal) No minimally imperfect graph has a star cutset.

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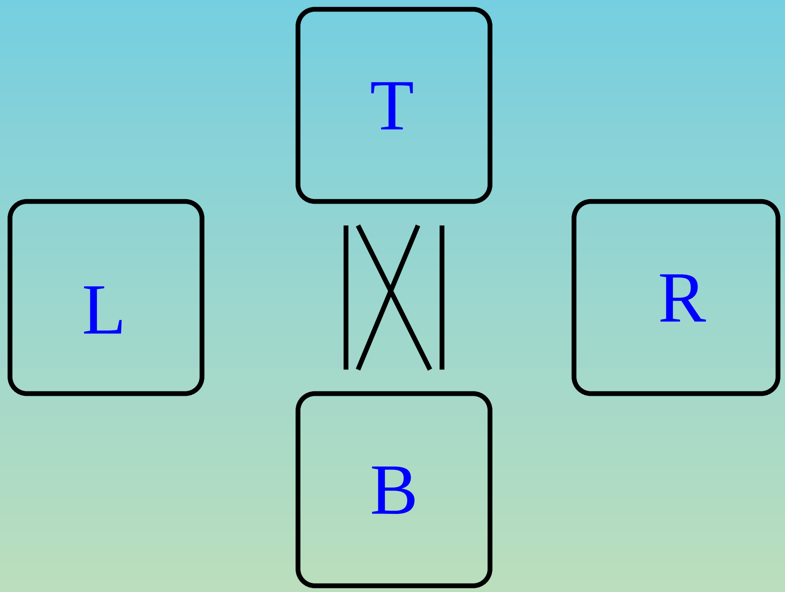

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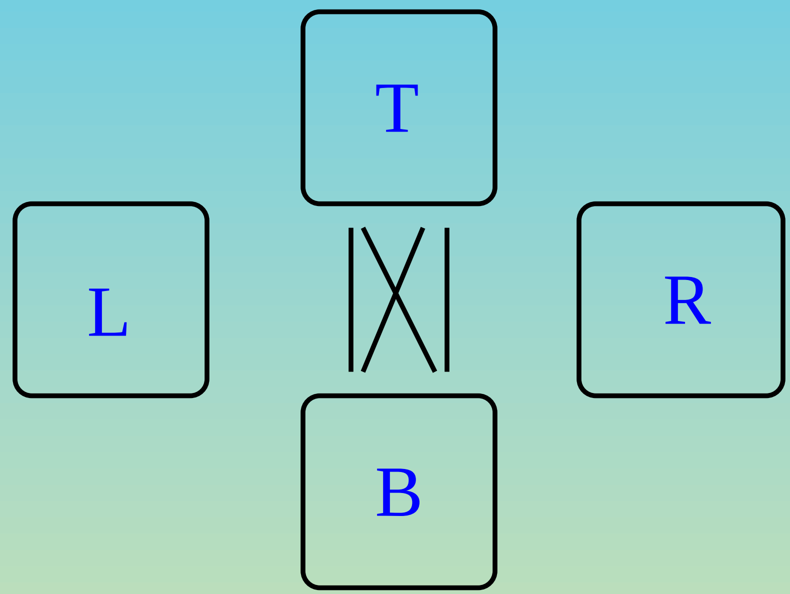

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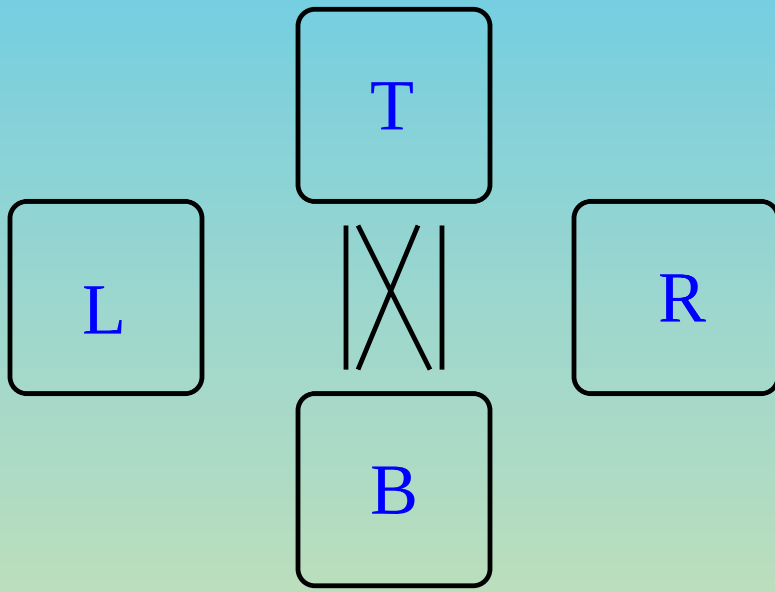

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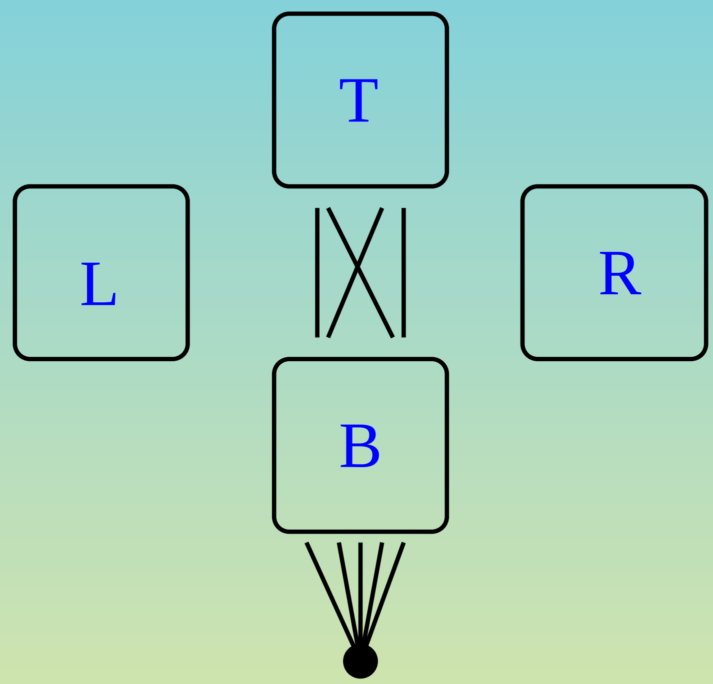
EVEN SKEW PARTITIONS

A skew partition is even if graph stays Berge after adding a vertex as shown:



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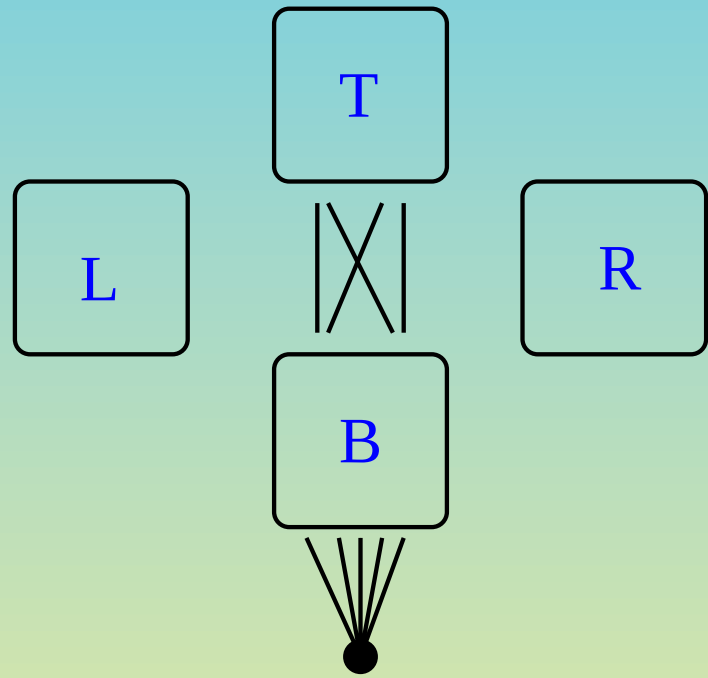

THM No minimum imperfect graph has an even skew partition.

MAIN THEOREM For every Berge graph G, either G or its complement

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(1) is bipartite, or

(2) is a line graph of a bipartite graph, or

(3) is a double split graph, or

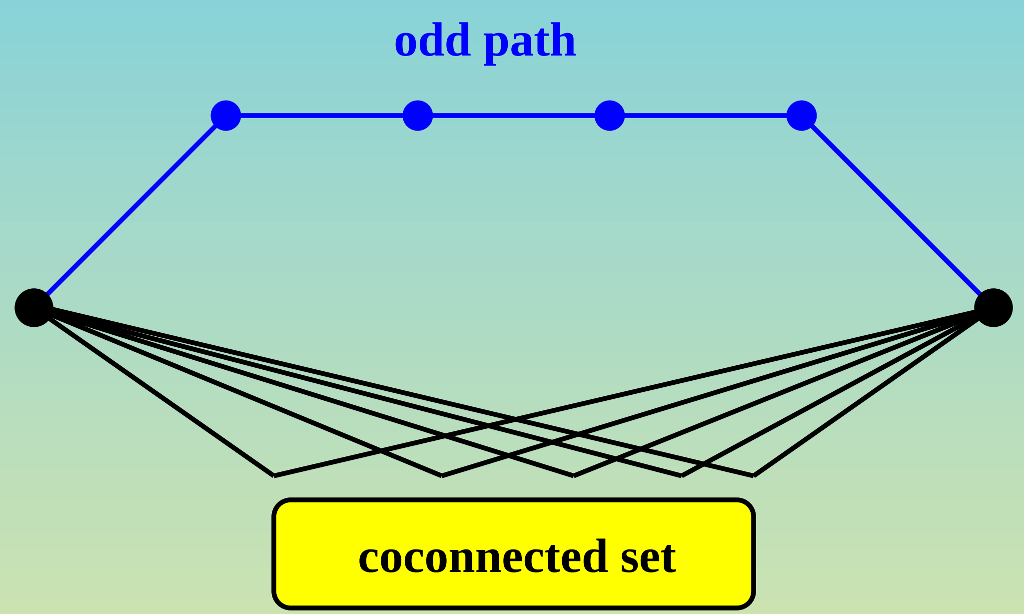
(4) has an even skew partition, or

(5) has a 2-join, or

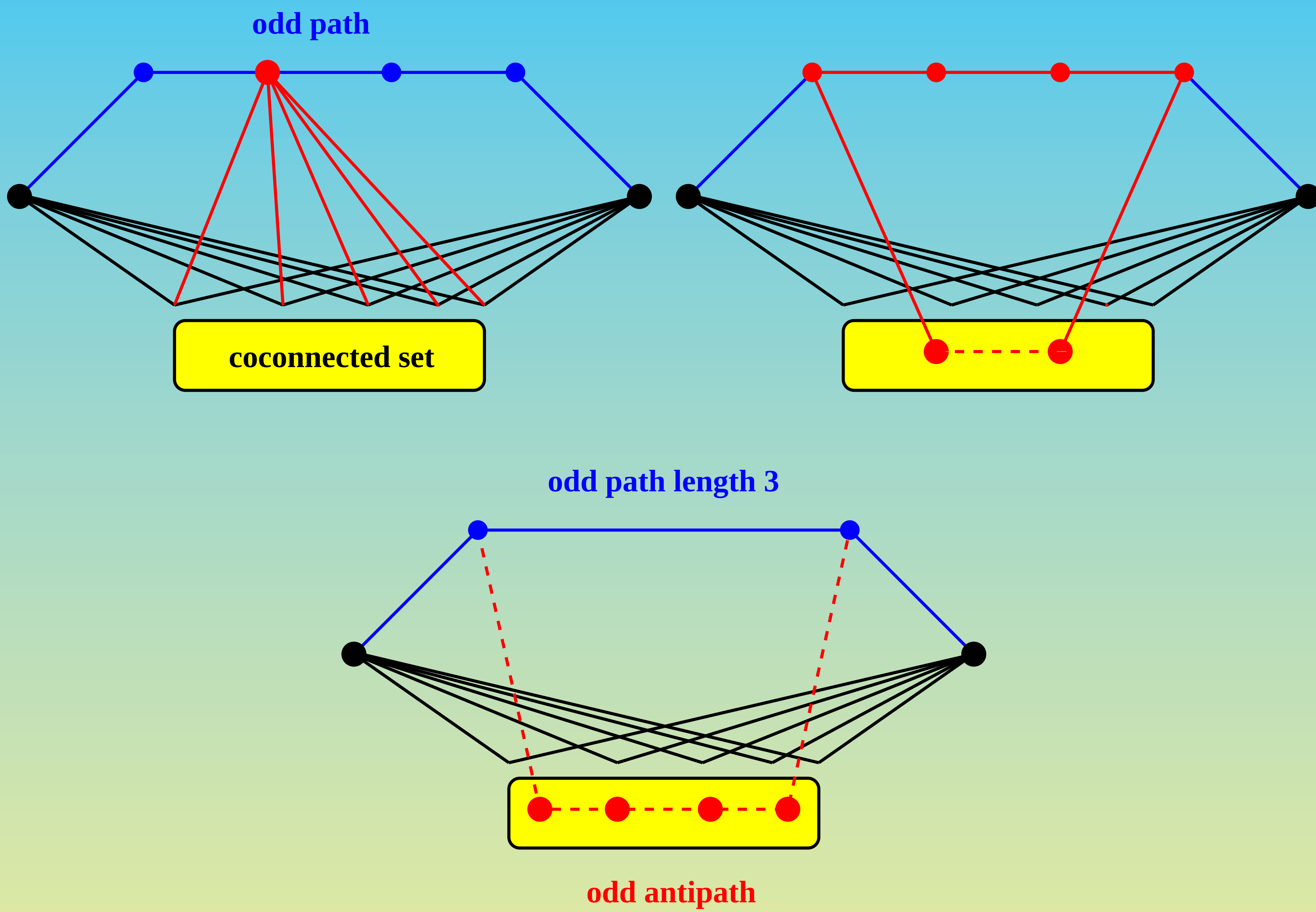
(6) has an M-join.

A LEMMA ABOUT ODD PATHS

Roussel & Rubio, RST In a Berge graph, if



then


odd path
coconnected set
odd path length 3
odd antipath

MAIN CASES

- G contains a "large" line graph of a bipartite graph
- G contains a "wheel"
- neither of the above

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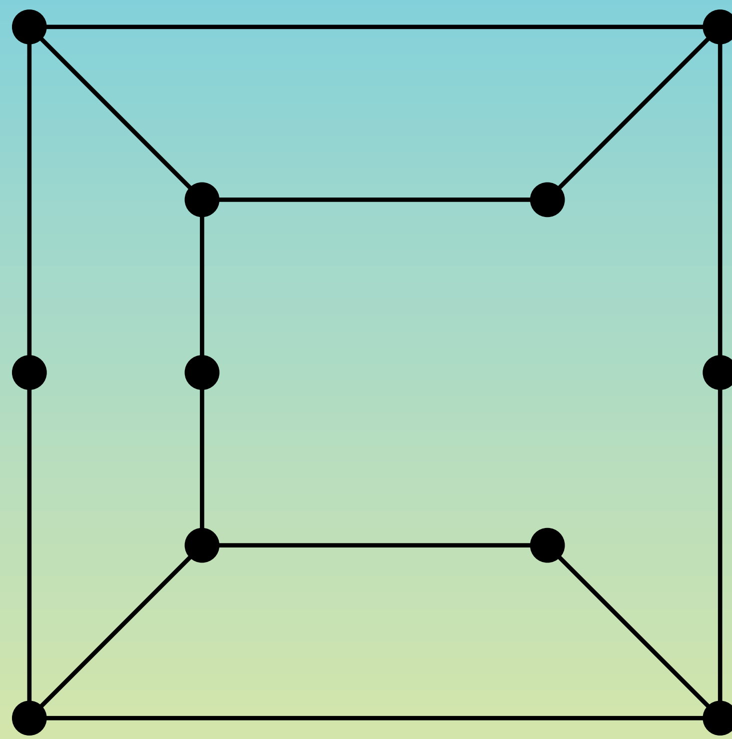

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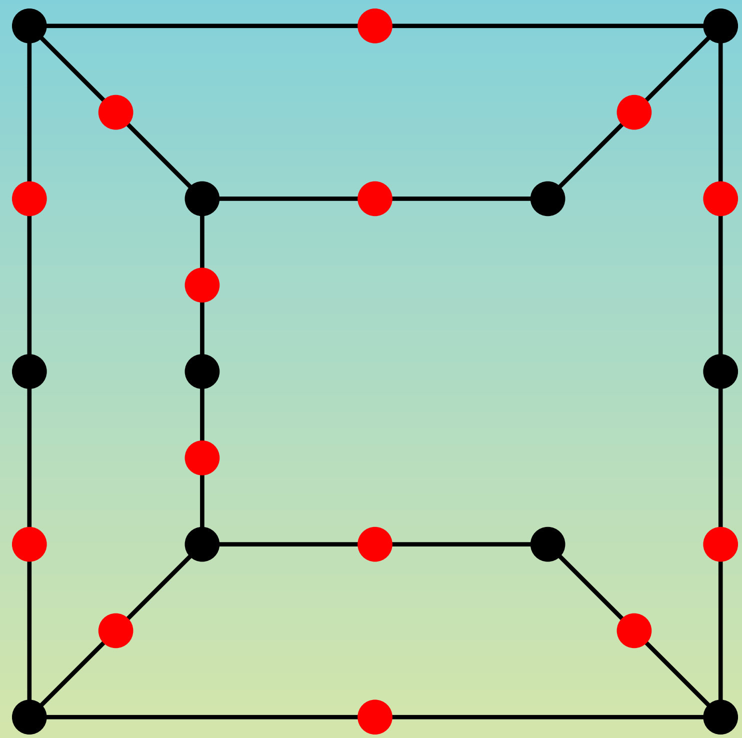

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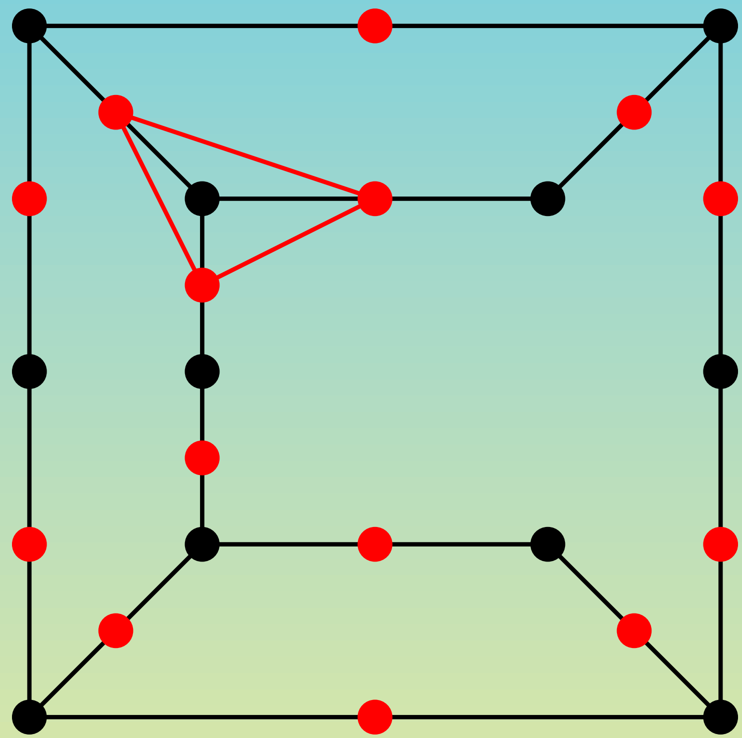

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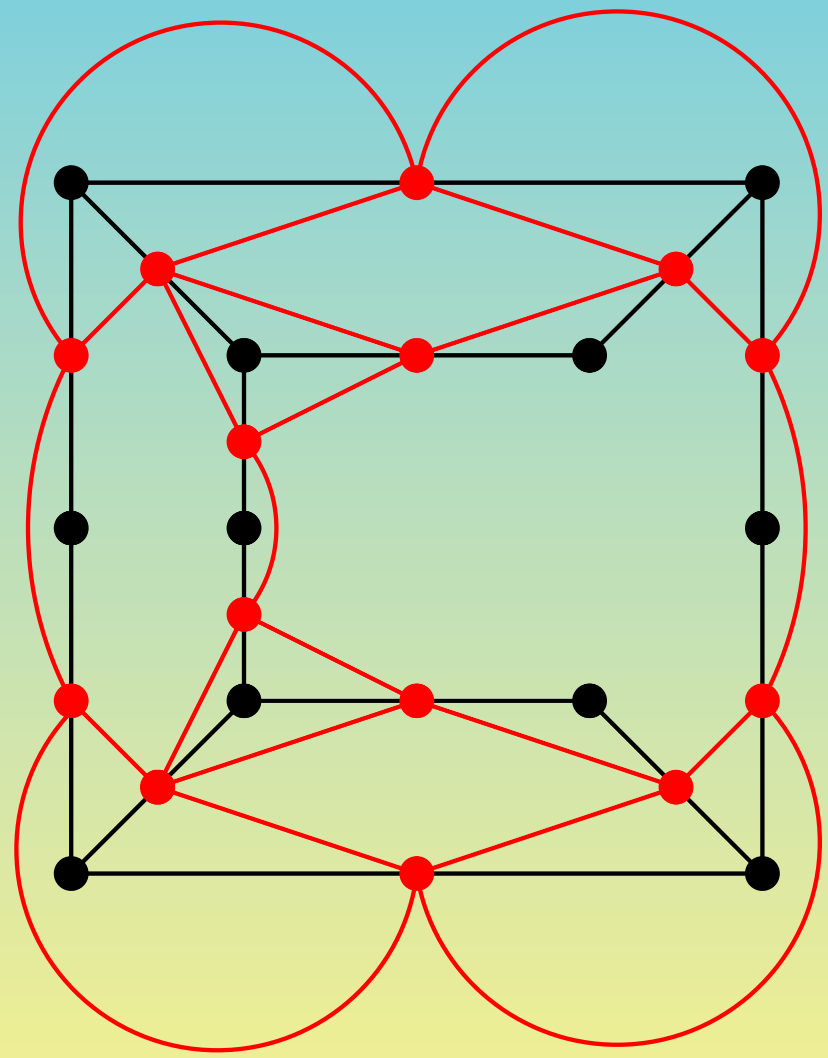

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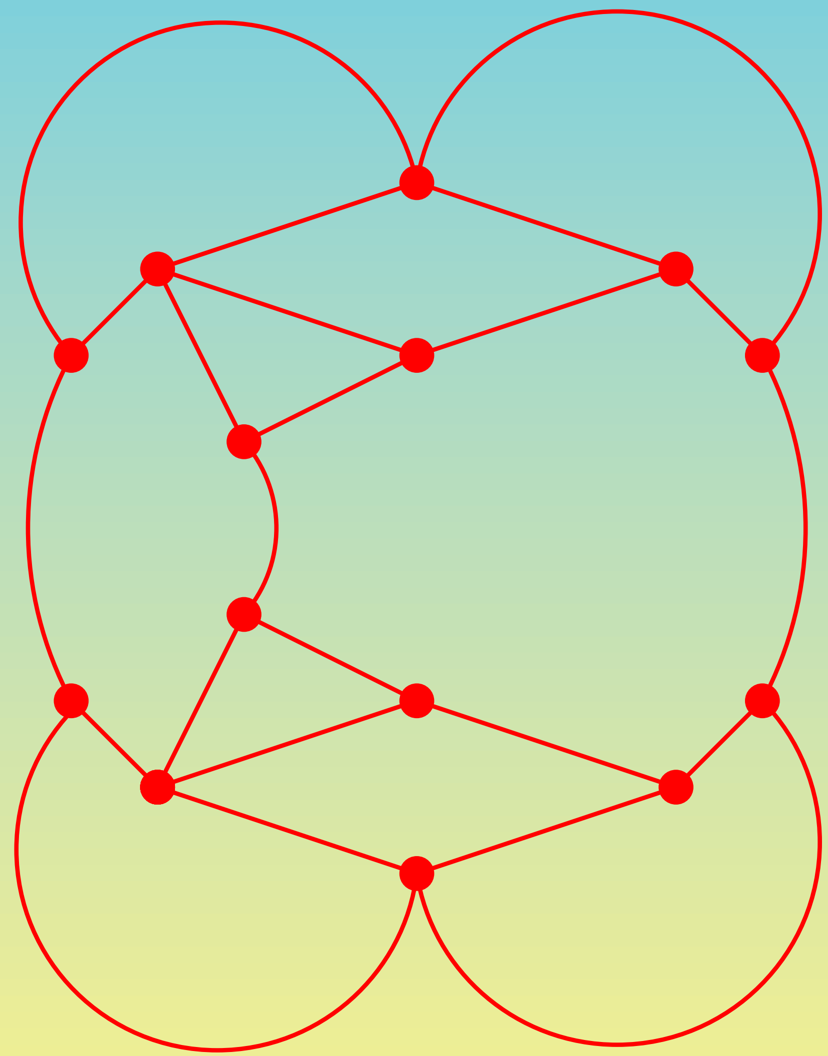

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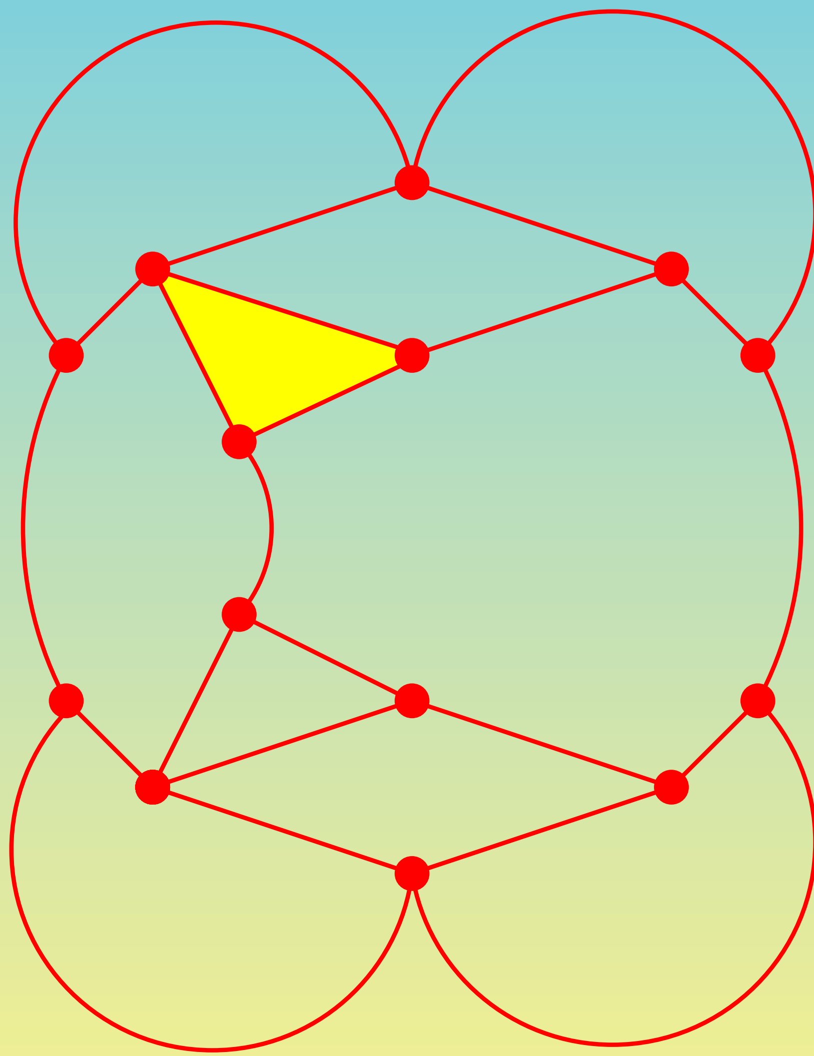
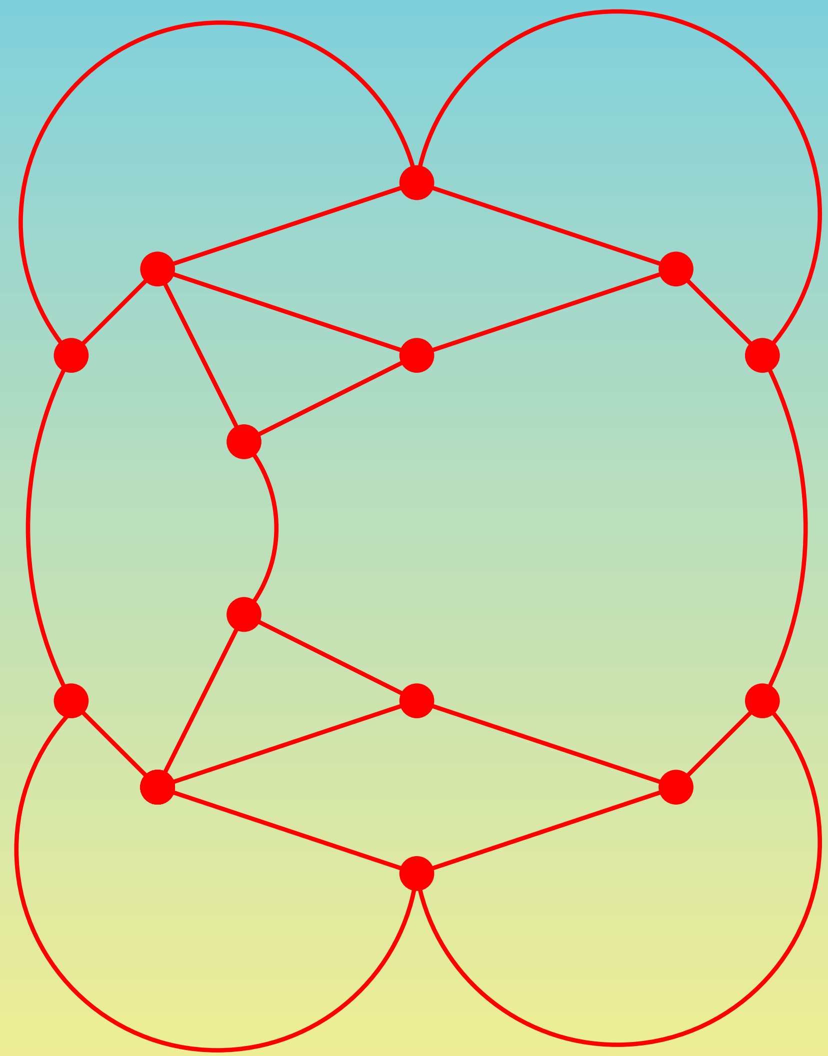
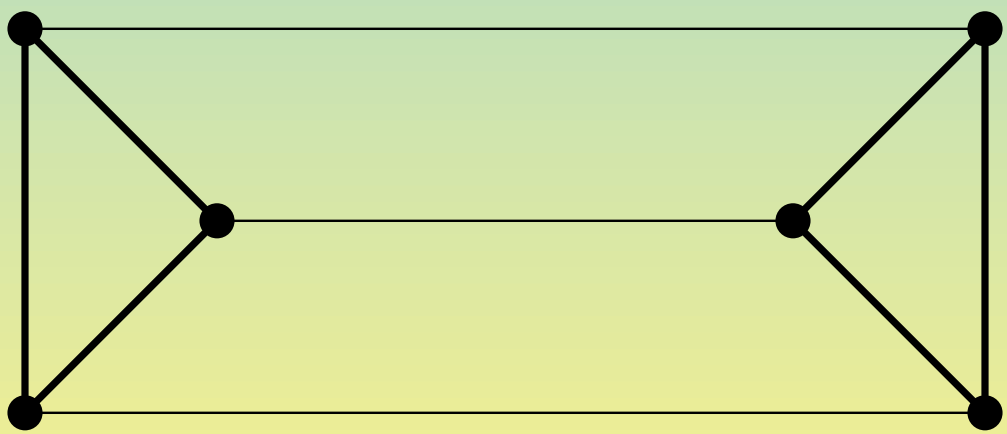


LINE GRAPHS



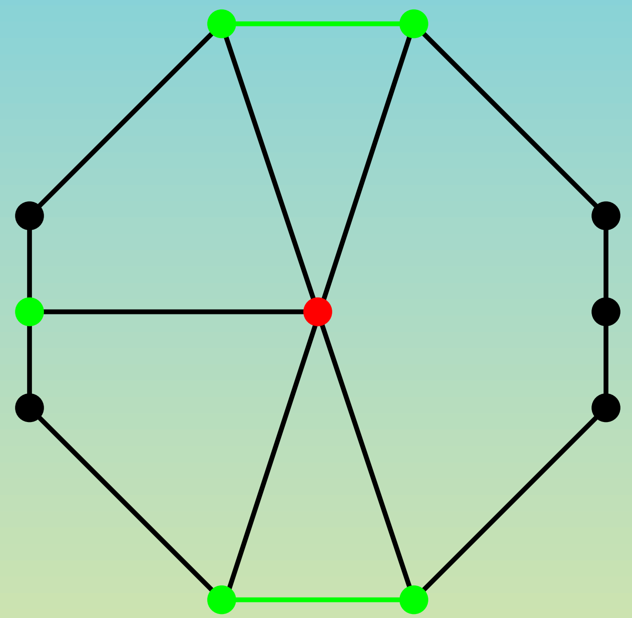
PRISMS

THEOREM If a Berge graph has a prism, then it or its complement is a line graph of a bipartite graph, is a double split graph, has skew partition, has a 2-join or has an M-join (and hence satisfies the conclusion of the main theorem).



SECOND STEP: WHEELS

A wheel consists of a hole of length ≥ 6 ("rim") and a vertex ("hub") forming ≥ 2 triangles.



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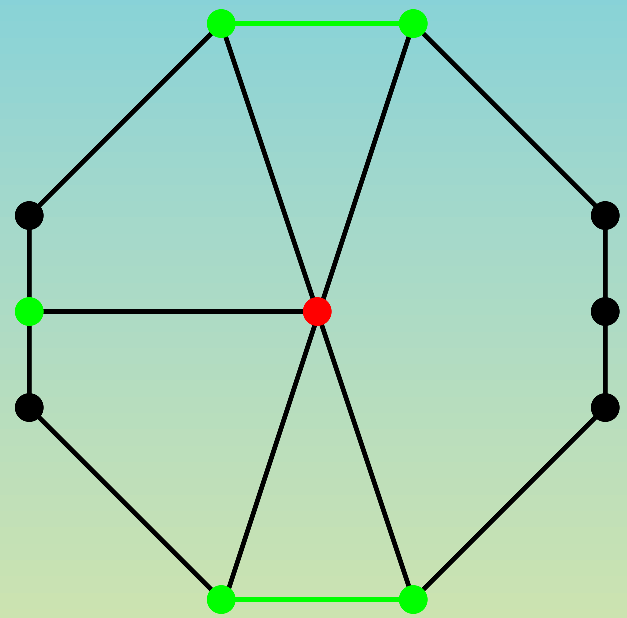
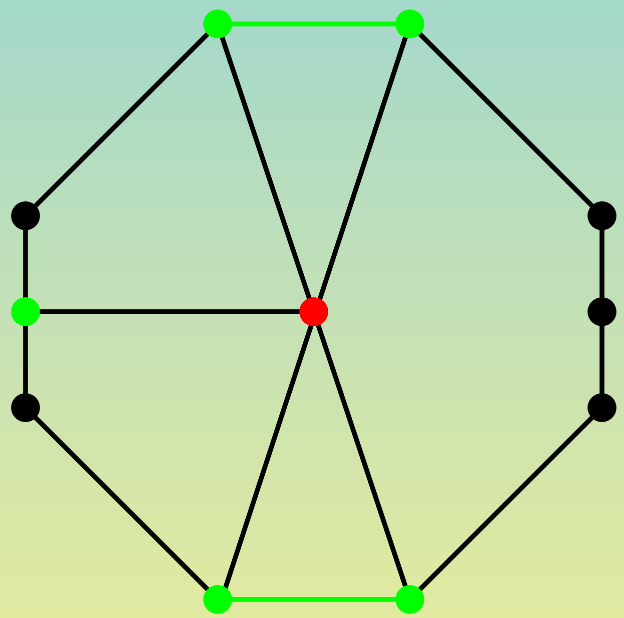


THEOREM If a Berge graph has a wheel, then it or its complement has a prism, a skew partition or a 2-join.

STRATEGY

Take a maximal coconnected set of hubs such that their common neighbors on the rim form ≥ 2 edges. Take all common neighbors of hubs. This tends to be a skew cutset.



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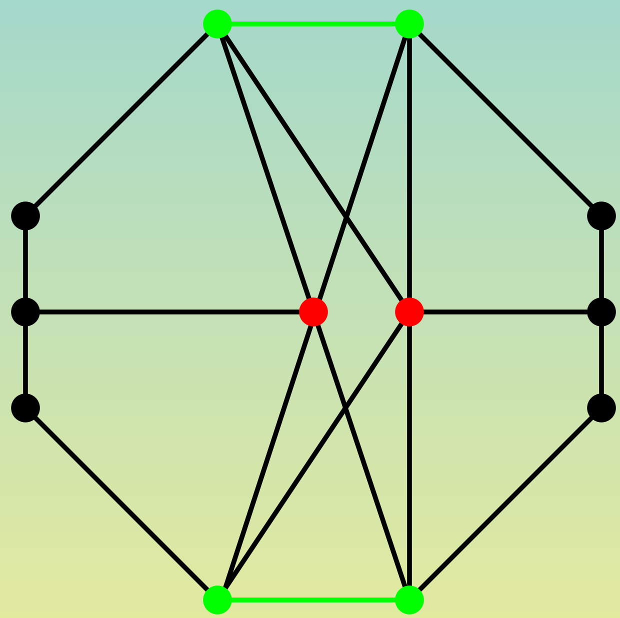

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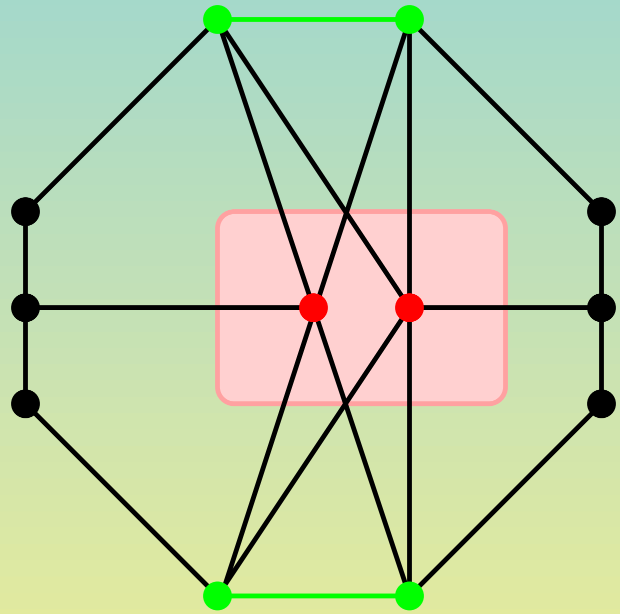

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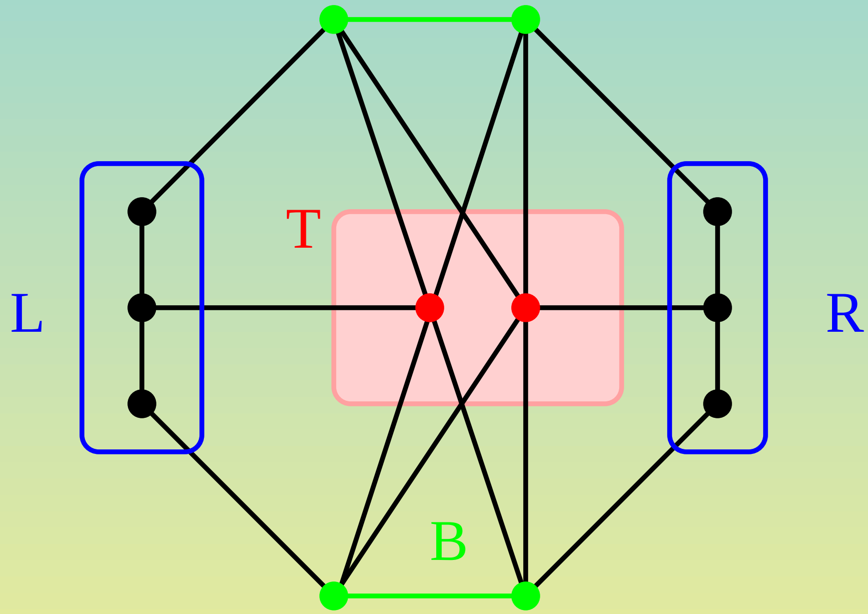
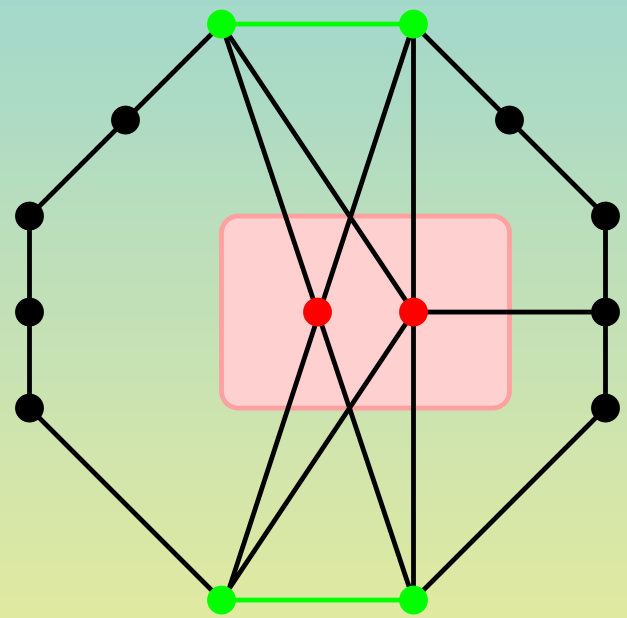


APPLICATION OF ODD PATH LEMMA

Every subpath of the rim between common neighbors of hubs is even or an edge.



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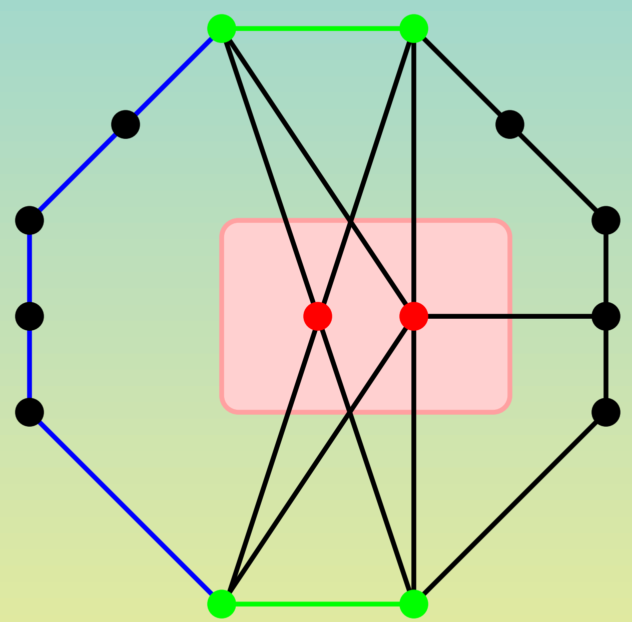

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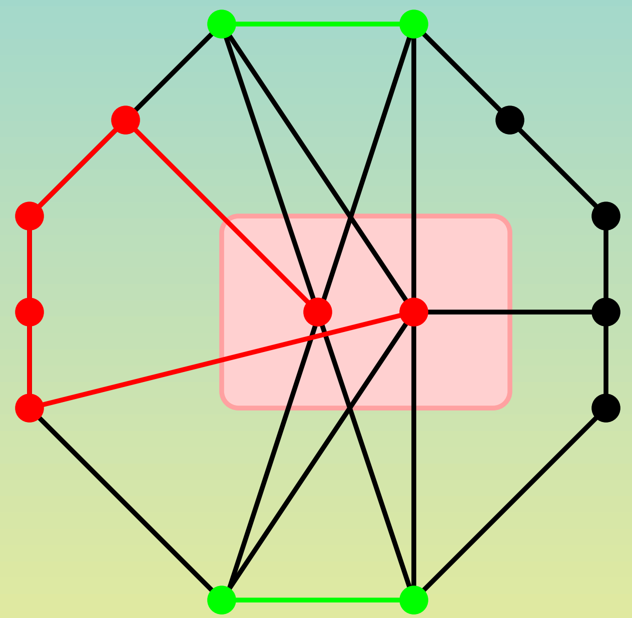

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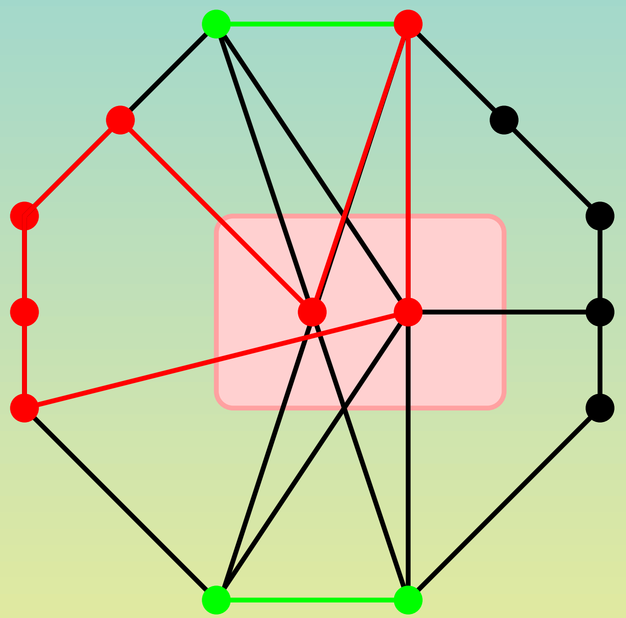

THIRD STEP

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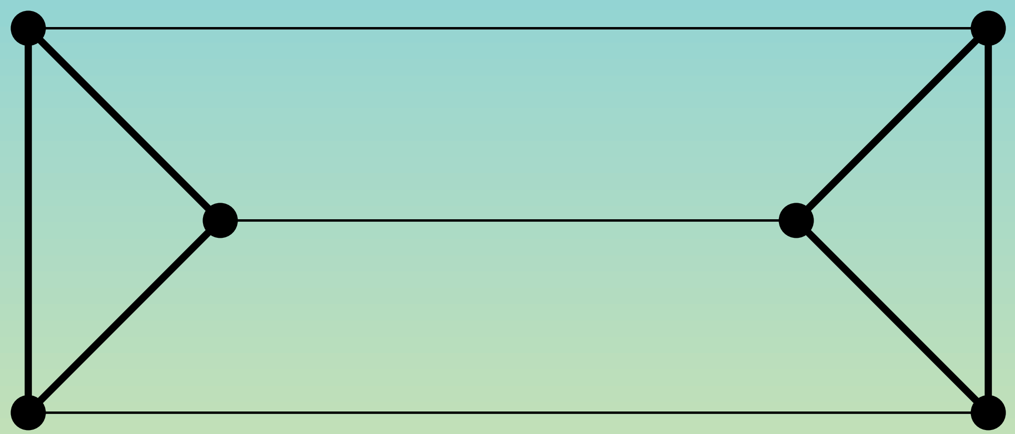

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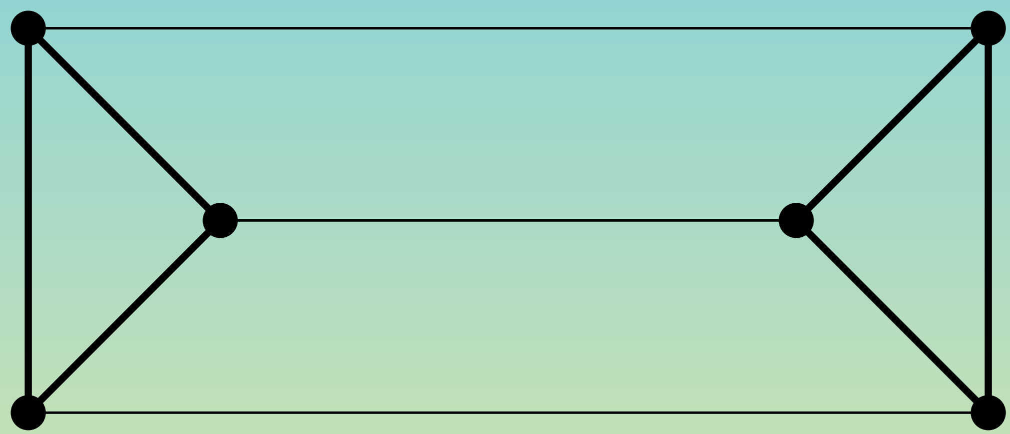

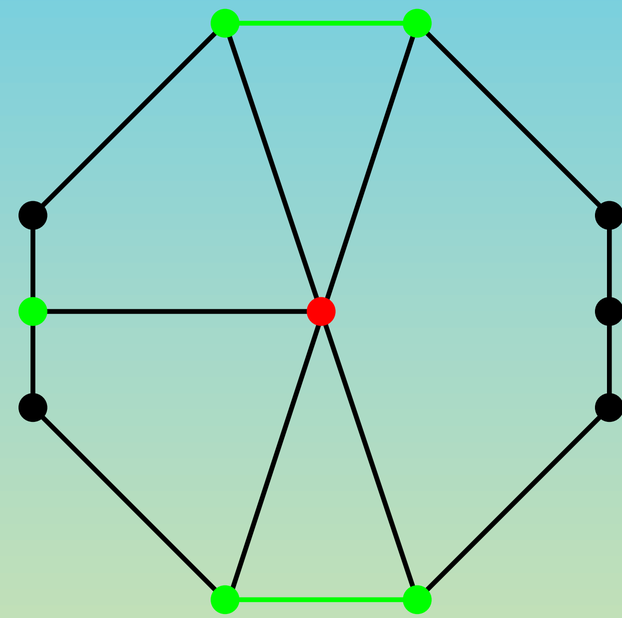

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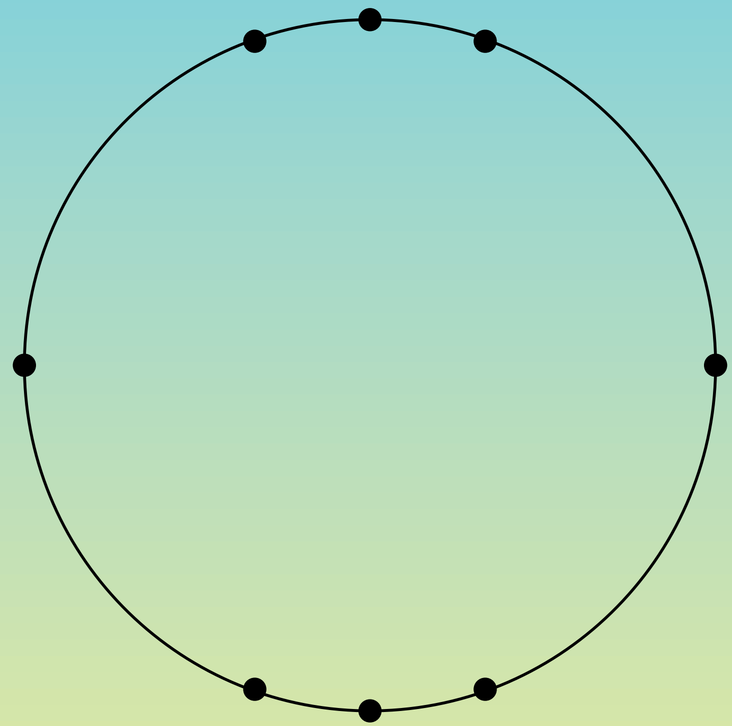

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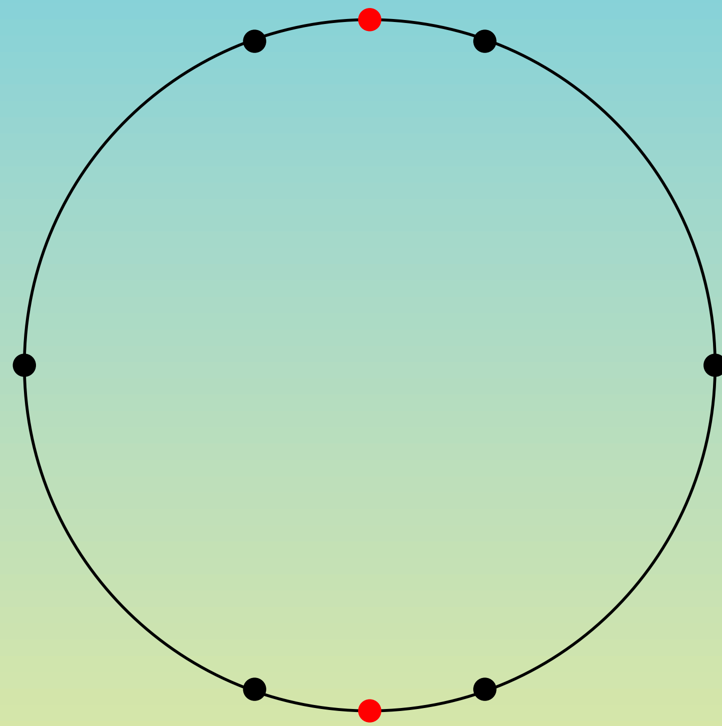

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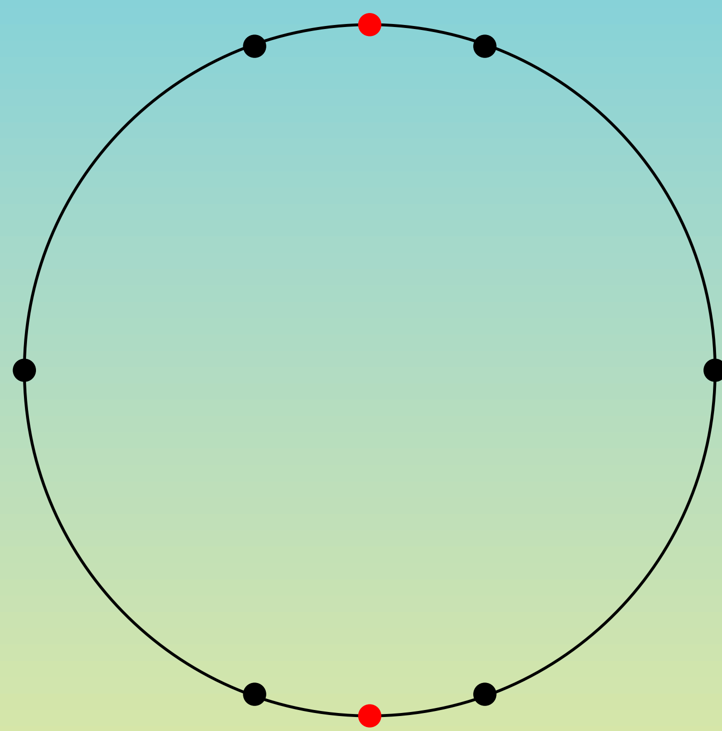

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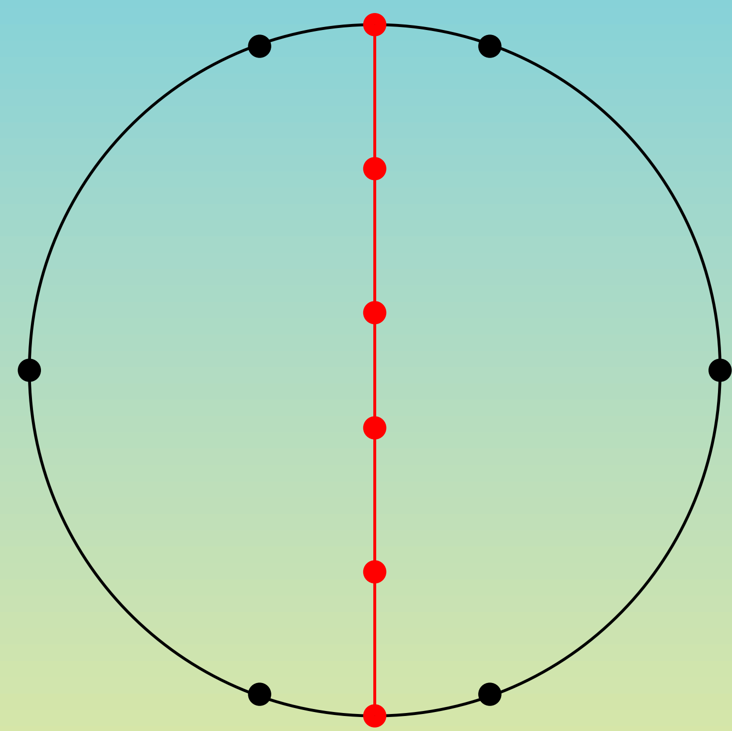

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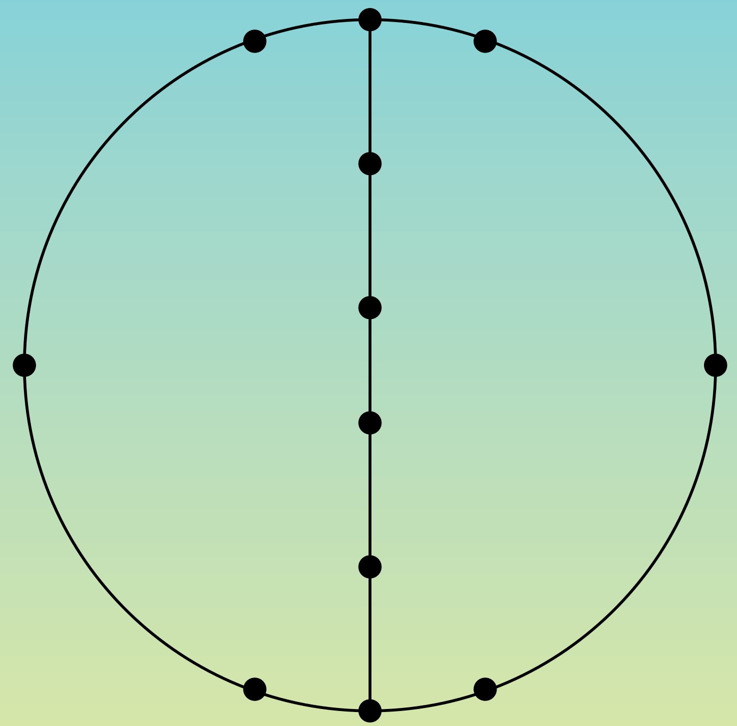

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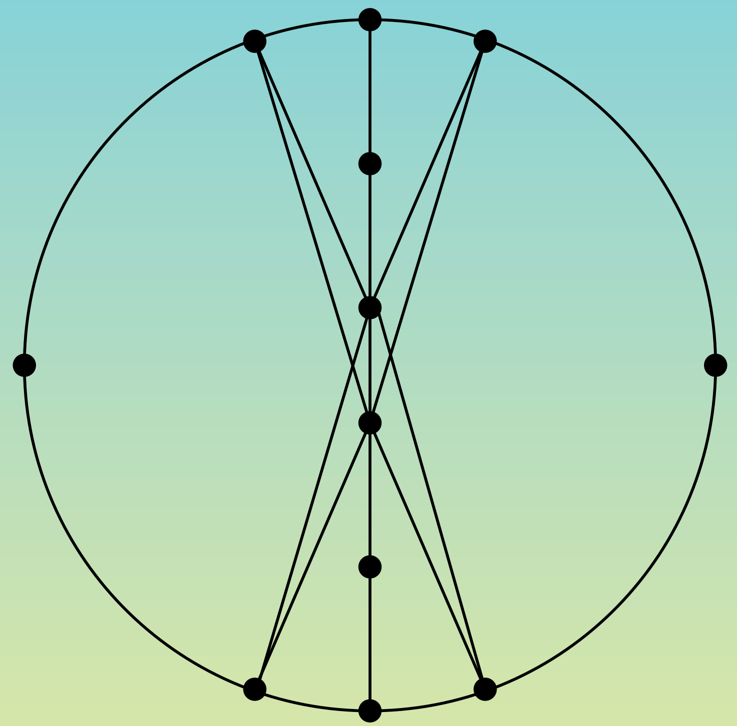

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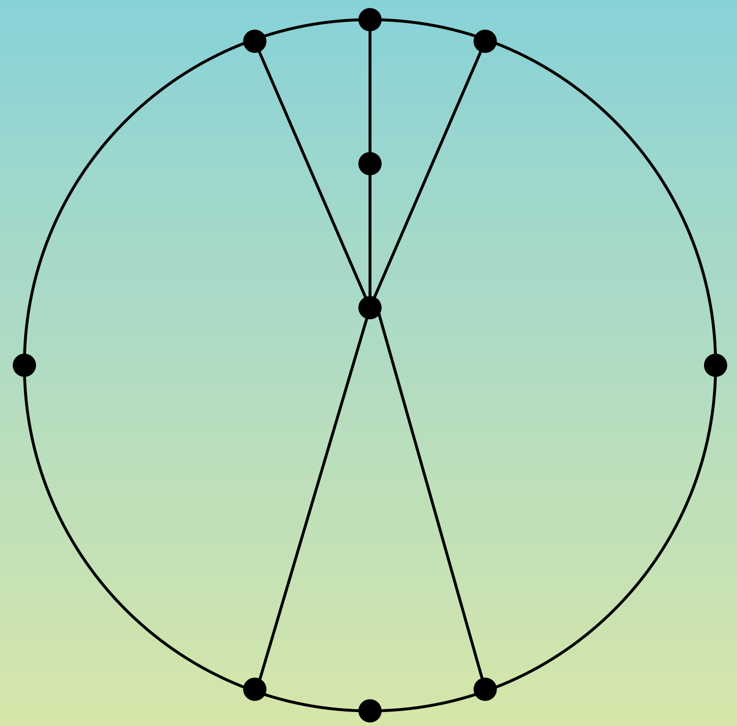

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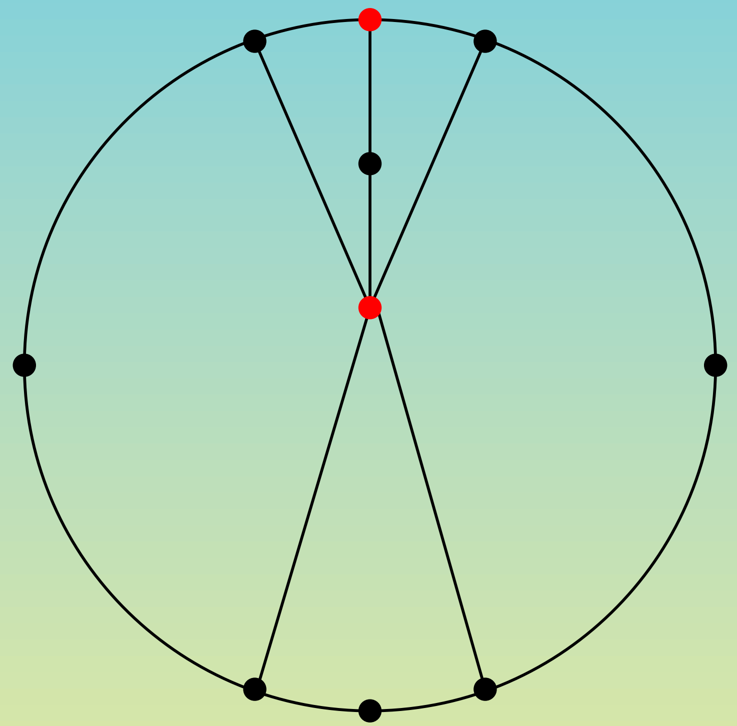

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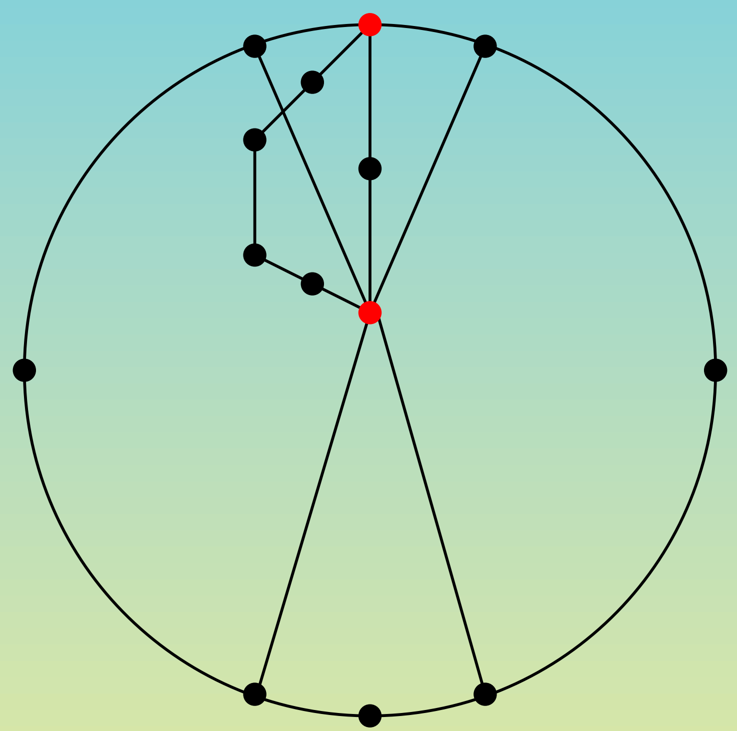

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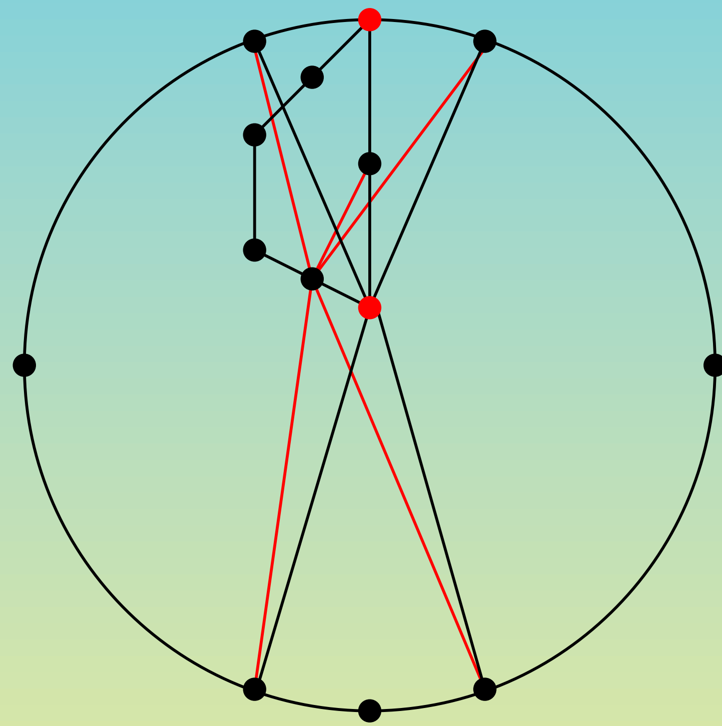

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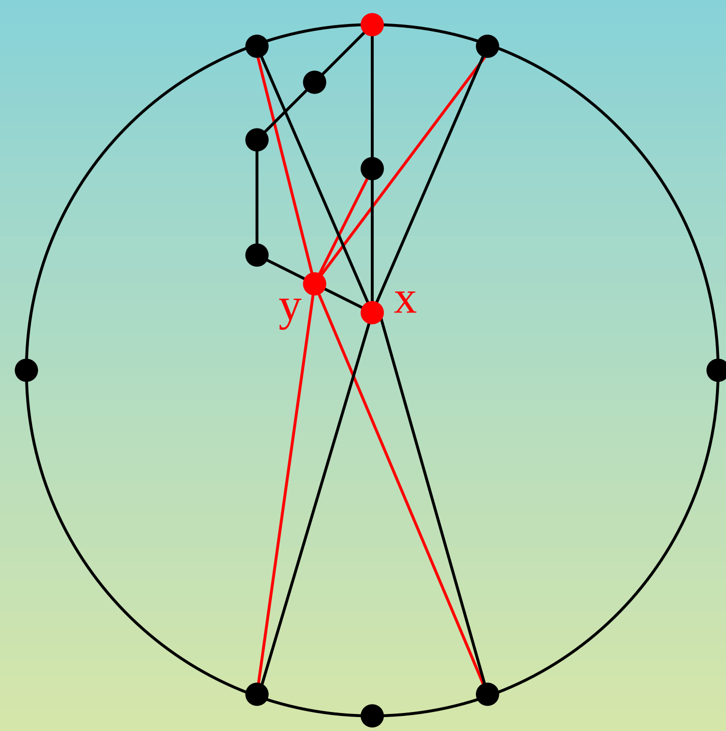

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