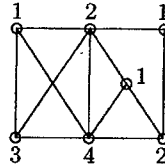
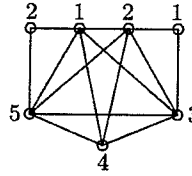


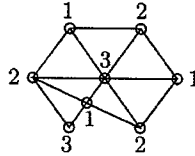
4. (a) A 4-coloring is shown; hence, $\chi(\mathcal{G}) \leq 4$.
 Since \mathcal{K}_4 is a subgraph, $\chi(\mathcal{G}) = 4$.



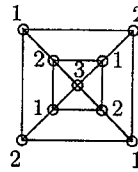
- (b) [BB] A 5-coloring is shown; hence, $\chi(\mathcal{G}) \leq 5$. Since \mathcal{K}_5 is a subgraph, $\chi(\mathcal{G}) = 5$.



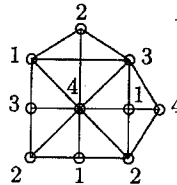
- (c) A 3-coloring is shown; hence, $\chi(\mathcal{G}) \leq 3$. Since \mathcal{K}_3 is a subgraph, $\chi(\mathcal{G}) = 3$.



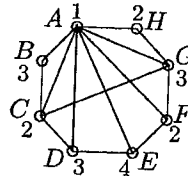
- (d) A 3-coloring is shown; hence, $\chi(\mathcal{G}) \leq 3$. Since \mathcal{G} contains \mathcal{K}_3 as a subgraph, $\chi(\mathcal{G}) = 3$.



- (e) [BB] A 4-coloring is shown; hence, $\chi(\mathcal{G}) \leq 4$. Since \mathcal{G} contains \mathcal{K}_4 as a subgraph, $\chi(\mathcal{G}) = 4$.



- (f) A 4-coloring is shown and, in fact, $\chi(\mathcal{G}) = 4$. Since ACG is a triangle, these vertices must have different colors. Since ACD and AFG are triangles, D and F must have different colors. But now a fourth color is needed for E , which is adjacent to A , D and F .



- 7 (c) The proof that $\chi(T) = 2$ is perhaps easiest by induction on n , the number of vertices. If $n = 2$, the tree must be $\circ - \circ$ and this can be colored with 2 colors, but not with fewer. Assuming the result for trees with k vertices, if we have a tree \mathcal{T} with $k + 1$ vertices, removing a vertex v of degree 1 (with the lone edge vw with which v is incident), we are left with a tree with k vertices. By the induction hypothesis, we can 2-color this tree. Then we assign to v the color not used for w and obtain a 2-coloring of \mathcal{T} .

24. Make a graph where vertices correspond to cities and an edge signifies that the corresponding cities are within 150 km and, therefore, must be assigned different channels. The number of channels required is the chromatic number of the graph which, in this problem, is five, as shown. (Note that the graph contains \mathcal{K}_5 ; vertices A, B, D, F and H .)

