Homework #1, 2602

Solutions:

3) Base Case: $n=32$

$32 = 3(9 \text{ pound fish}) + 1(5 \text{ pound fish})$ ✓

IH: There exists some combination $9x + 5y = n$ for $n \geq 32$ w/ $x \geq 0, y \geq 0$

IS: Show there exists, x', y' w/ $9x' + 5y' = n+1$.

Case 1: $x \geq 1 \Rightarrow$

$9(x-1) + 5(y+2) = n - 9 + 10 = n+1 \Rightarrow$

$x-1 = x'$, $y+2 = y'$ work.

Case 2: $x=0 \Rightarrow$ since $n \geq 32$ we know $y \geq 7 \Rightarrow$

$9 \cdot 4 + 5(y-7) = n + 36 - 35 = n+1 \Rightarrow$

$x' = 4$, $y' = y-7$ work.

4) c) Base Case: $n=1$,

$5^{(1)}-1=4$ and $4/4$.

I.H. $4/ 5^n-1$

I.S. Prove $4/ 5^{n+1}-1$

$5^{n+1}-1 = 4\cdot 5^n + (5^n-1)$

$4/(4\cdot 5^n)$ and $4/(5^n-1)$ by the I.H. $\Rightarrow$

$4/ 4 5^n + 5^n - 1$ proving the claim.

4) g) Base Case: $n=1$

$1^5 + 5\cdot 1 = 6$, $6/6$ ✓

I.H.: $6/ n^3+5n$

I.S.: Prove $6/ (n+1)^3 + 5(n+1)$

$(n+1)^3 + 5(n+1) = (n^3+3n^2+3n+1) + 5n+5 =$

$(n^3+5n) + (3n^2+3n) + 6$

$5/ n^3+5n$ by the I.H. and $6/6$ $\Rightarrow$

$6/ (n+1)^3+5(n+1)$ if and only if

$6/ 3n^2+3n$, $3n^2+3n = 3n(n+1)$

$\Rightarrow 6/ 3n^2+3n$ if and only if

$2/ n(n+1)$. $2/ n(n+1)$ as either n or $n+1$ is even.

6) b) Base Case: $n=1$

$$1^2 = (-1)^{1-1}\frac{(1)(1+1)}{2} = 1 \checkmark$$

I.H.: $\sum_{i=1}^{n}(-1)^{i-1}i^2 = (-1)^{n-1}\frac{n(n+1)}{2}$

I.S.: Prove: $\sum_{i=1}^{n+1}(-1)^{i-1}i^2 = (-1)^{(n+1)-1}\frac{(n+1)((n+1)+1)}{2} = \frac{(-1)^n(n+1)(n+2)}{2}$.

$$\sum_{i=1}^{n+1}(-1)^{i-1}i^2 = \sum_{i=1}^{n}(-1)^{i-1}i^2 + (-1)^{(n+1)-1}(n+1)^2 =$$

$$(\text{by I.H.}) = (-1)^{n-1}\frac{n(n+1)}{2} + (-1)^n(n+1)^2 =$$

$$(-1)^{n-1}\left[\frac{n(n+1)}{2} - (n^2+2n+1)\right] =$$

$$(-1)^{n-1}\left(\frac{n^2+n-2n^2-4n-2}{2}\right) = (-1)^{n-1}\left(\frac{-n^2-3n-2}{2}\right)$$

$$= \frac{(-1)^n(n^2+3n+2)}{2} = \frac{(-1)^n(n+1)(n+2)}{2} \checkmark$$

9)g Base Case: $n=2$

$$\frac{1}{1^2}+\frac{1}{2^2}=\frac{5}{4}<2-\frac{1}{2}=\frac{3}{2}=\frac{6}{4}\ \checkmark$$

I.H.: $\sum_{i=1}^{n}(i)^{-2}<2-\frac{1}{n}$ for $n\geq 2$

I.S.: Prove $\sum_{i=1}^{n+1} i^{-2}<2-\frac{1}{n+1}$

$$\sum_{i=1}^{n+1} i^{-2}=\sum_{i=1}^{n} i^{-2}+\frac{1}{(n+1)^2}<\text{(by I.H.)}$$

$$\left(2-\frac{1}{n}\right)+\frac{1}{(n+1)^2}\leq 2-\frac{1}{n+1}$$ if and only if

$$\frac{1}{(n+1)^2}-\frac{1}{n}\leq\frac{-1}{n+1}\iff$$

$$\frac{n(n+1)^2}{(n+1)^2}-\frac{n(n+1)^2}{n}\leq\frac{(-1)(n+1)^2 n}{n+1}\iff$$

$$n-(n+1)^2\leq-(n+1)n\iff$$

$$-n^2-n-1\leq-n^2-n$$ which is true for $n\geq 2$.