27. $a_n - 3a_{n/2} = n+1$, n a power of two

Let $n = 2^k$ and $b_k = a_{2^k}$. Then

(*) $$b_k - 3b_{k-1} = 2^k + 1$$

Homogeneous equation has solution $b_k^H = A\,3^k$.

We need to find a particular solution to

$$b_k^{I} - 3b_{k-1}^{I} = 2^k \quad \text{and} \quad b_k^{II} - 3b_{k-1}^{II} = 1$$

Look in the form

$$b_k^{I} = B2^k \qquad b_k^{II} = C$$

Substitute into recurrence relation

$$B\cdot 2^k - 3B2^{k-1} = 2^k \qquad C - 3C = 1$$

$$2B - 3B = 2 \qquad C = -1/2$$

$$B = -2$$

So the general solution to (*) is

$$b_k = b_k^H + b_k^{I} + b_k^{II} = A3^k - 2\cdot 2^k - 1/2$$

and so

$$a_n = A\,3^k - 2n - 1/2 = A\left(2^{\log_2 3}\right)^k - 2n - 1/2 =$$

$$= A\left(2^k\right)^{\log_2 3} - 2n - 1/2 = A\,n^{\log_2 3} - 2n - 1/2$$

28. Use $n = 3^k$ and $b_k = a_{3^k}$

We get $a_n = (\log_3 n + A)\,n - 1/2$