

About roots and multiplicities

Consider the equation

$$(1) \quad x^2 + c_1x + c_2 = 0.$$

The Fundamental Theorem of Algebra tells us that the above equation has exactly two roots, say x_1, x_2 , but they could be equal. If $x_1 \neq x_2$, then we say that x_1 and x_2 are roots of equation (1) of *multiplicity one*. If $x_1 = x_2$, then $x_1 = x_2$ is a root of *multiplicity two*. Any number other than x_1, x_2 is a root of *multiplicity zero*.

Example. $x^2 - 3x + 2 = 0$

Since $x^2 - 3x + 2 = (x - 2)(x - 1) = 0$, this equation has two roots $x_1 = 1, x_2 = 2$, both of multiplicity one.

Example. $x^2 - 6x + 9 = 0$

$x^2 - 6x + 9 = (x - 3)^2 = 0$. This equation has one root $x = 3$ of multiplicity two.