

NAME: _____

Math 2602

Test 1

October 21, 2009

TA:

Instructions. Please write your name legibly on top of each page. Closed book, closed notes. Small nonprogrammable calculators are allowed. To receive credit you must show your work and justify your answers. Please circle or box your answer. Each question is worth ten points.

A pizza parlor offers the following toppings for its pizza: sausage, anchovies, onions, olives, mushrooms. Any combination of zero or more toppings may be ordered (no doubles).

1. How many different pizzas can be ordered?

$$2^5 = \textcircled{32}$$

2. How many different pizzas with at least two toppings can be ordered?

$$\begin{array}{ccccccccccc} \binom{5}{2} & + & \binom{5}{3} & + & \binom{5}{4} & + & \binom{5}{5} & = & 10 & + & 10 & + & 5 & + & 1 & = & \textcircled{26} \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & & & & & & & & & \\ 2 \text{ toppings} & & 3 \text{ toppings} & & 4 \text{ toppings} & & 5 \text{ toppings} & & & & & & & & & & \end{array}$$

3. If a pizza is ordered at random, what is the probability that it will have an odd number of toppings? Assume all pizzas are equally likely.

$$\frac{\binom{5}{1} + \binom{5}{3} + \binom{5}{5}}{2^5} = \frac{5 + 10 + 1}{32} = \frac{16}{32} = \textcircled{\frac{1}{2}}$$

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4. Five identical fair coins fall on the floor. What is the probability of exactly two heads?

$$\frac{\binom{5}{2}}{2^5} = \frac{10}{32} = \frac{5}{16}$$

5. How many arrangements are there of the letters of the word MISSISSIPPI?

4 × S
4 × I
2 × P
1 × M

$$\binom{11}{4, 4, 2, 1} = \frac{11!}{4! 4! 2!} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 2} =$$

$$= 11 \cdot 10 \cdot 9 \cdot 7 \cdot 5 = 34,650$$

6. Find the coefficient of x^3 in $(3x - 2)^6$.

$$\binom{6}{3} (3x)^3 (-2)^3 = \frac{6 \cdot 5 \cdot 4}{6} 6^3 x^3 = -216 \cdot 20x^3 = -4320x^3$$

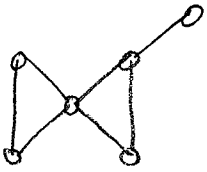
$$-4320$$

7. Define what it means for a graph to be Eulerian.

A graph is Eulerian if it has a closed walk that uses every edge exactly once.

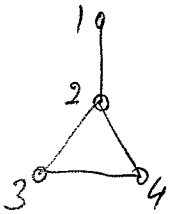
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8. Is the following graph Eulerian? Explain.



No. Not every vertex has even degree

9. Let A be the adjacency matrix of the following graph. Find the $(1, 2)$ entry of A^4 .



The $(1, 2)$ entry of A^4 is equal to the number of walks from 1 to 2 of length four. There are exactly two: 12342 and 12432

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10. In how many ways can five red balls, six blue balls and seven green balls be placed in twenty boxes numbered $1, 2, \dots, 20$?

$$\binom{20+5-1}{5} \binom{20+6-1}{6} \binom{20+7-1}{7} =$$
$$= \frac{24! \cdot 25! \cdot 27!}{5! \cdot 19! \cdot 6! \cdot 19! \cdot 7! \cdot 19!}$$