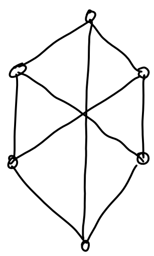
NAME: Key

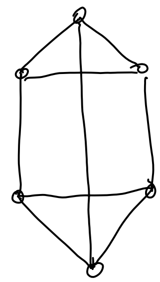
Math 2602 Test 1 November 18, 2009 TA:

Instructions. Please write your name legibly on top of each page. Closed book, closed notes. Small nonprogrammable calculators are allowed. To receive credit you must show your work and justify your answers. Please circle or box your answer. Good luck!

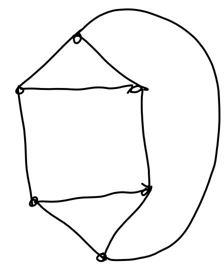
1. (10 points) For each of the graphs below determine whether it is planar. If it is planar, then draw an isomorphic plane graph. If it is not planar, then explain why it is not.



No, this graph is $K_{3,3}$



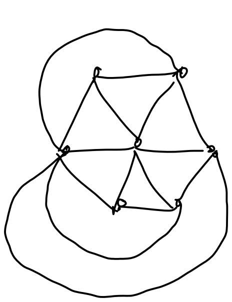
Yes:



2. (10 points) Determine the maximum number m of edges a planar graph on 7 vertices can have. Prove that no planar graph on 7 vertices can have $m+1$ or more edges, and exhibit a planar graph on 7 vertices and m edges.

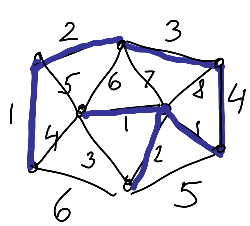
In a planar graph $m \leq 3|V(G)| - 6 = 3 \cdot 7 - 6 = 15$.

Here is a planar graph on 7 vertices and 15 edges.



$m = 15$

3. (15 points) Use Prim's algorithm to find a minimum weight spanning tree in the weighted graph below. Explain one step of the algorithm in detail.



Suppose we have constructed a tree T. We pick an edge of least weight that joins a vertex in T to a vertex not in T and add it to T.

4. (20 points) Use the simplex algorithm to solve the following problem: Maximize $2x_1 + 3x_2$ subject to $x_1 + x_2 \leq 3$, $x_1 + 2x_2 \leq 4$, $x_1 \geq 0$, $x_2 \geq 0$. At each step circle the pivot cell.

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & S_1 & S_2 & \end{array} \\ \begin{array}{c} \\ S_1 \\ S_2 \end{array} & \begin{pmatrix} -2 & -3 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 3 \\ 1 & 2 & 0 & 1 & 4 \end{pmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & S_1 & S_2 & \end{array} \\ \begin{array}{c} \\ S_1 \\ x_2 \end{array} & \begin{pmatrix} -1/2 & 0 & 0 & 3/2 & 6 \\ 1/2 & 0 & 1 & -1/2 & 1 \\ 1/2 & 1 & 0 & 1/2 & 2 \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccccc} x_1 & x_2 & S_1 & S_2 & \end{array} \\ \begin{array}{c} \\ x_1 \\ x_2 \end{array} & \begin{pmatrix} 0 & 0 & 1 & 1 & 7 \\ 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & -1 & 1 & 1 \end{pmatrix} \end{array}$$

Maximum of 7 attained for $x_1 = 2, x_2 = 1$.

5. (10 points) Write the following linear program in standard form (but do not solve): Minimize $x_1 + 2x_2 - 4x_3$ subject to $2x_1 + x_2 = 3$, $3x_1 - x_2 + x_3 \leq 5$, $x_1 \geq 0$, $x_2 \geq 0$.

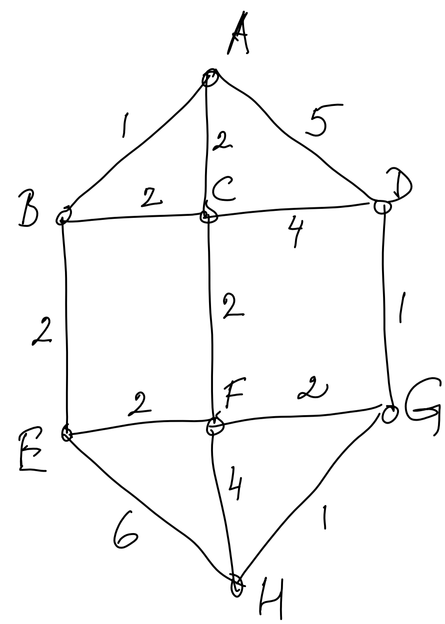
$$\begin{aligned} \max \quad & x_1 + 2x_2 - 4y_3 + 4z_3 \\ \text{s.t.} \quad & 2x_1 + x_2 = 3 \\ & 3x_1 - x_2 + y_3 - z_3 + S = 5 \\ & x_1, x_2, y_3, z_3, S \geq 0 \end{aligned}$$

6. (10 points) Prove or disprove: If G is a graph on 100 vertices and every vertex has degree at least 45, then G is Hamiltonian.

Not true. Take a graph G consisting of two disjoint copies of K_{50}. It has 100 vertices, every vertex has degree 49, but it is not Hamiltonian, because it is not connected.

7. (15 points) Use Dijkstra's algorithm to find the length of a shortest path from A to every other vertex of the weighted graph below. For every step of the algorithm, write the label of each vertex in the matrix below. Circle the label when it becomes permanent.

	1	2	3	4	5	6	7	8
A	0	0	0	0	0	0	0	0
B	∞	1	1	1	1	1	1	1
C	∞	2	2	2	2	2	2	2
D	∞	5	5	5	5	5	5	5
E	∞	∞	3	3	3	3	3	3
F	∞	∞	∞	4	4	4	4	4
G	∞	∞	∞	∞	∞	6	6	6
H	∞	∞	∞	∞	9	8	8	7



8. (10 points) Prove or disprove: If $\chi(G) \leq 4$, then G is planar.

Not true. $\chi(K_{3,3}) = 2$, and yet $K_{3,3}$ is not planar.