WEEK 13 PROBLEMS

Math 6014A

1. A graph is *outerplanar* if it is isomorphic to a plane graph such that every vertex is incident with the unbounded face. Prove that a graph is outerplanar if and only if it has no subgraph isomorphic to a subdivision of K_4 or $K_{2,3}$.

2. Prove that a 2-connected plane graph is bipartite if and only if every face is bounded by an even cycle.

3. Characterize graphs H such that for every graph G the following holds: G has a minor isomorphic to H if and only if G has a subgraph isomorphic to a subdivision of H.

4. A plane graph is a *triangulation* if every face is bounded by a cycle of length three. Prove that a loopless plane triangulation G has chromatic number 3 if and only if every vertex of G has even degree.

5. Prove that the faces of a Hamiltonian plane graph can be 4-colored in a such a way that whenever two faces are incident with the same edge they receive different colors.

6. Prove that a graph has a K_5 or $K_{3,3}$ subdivision if and only if it has a K_5 or $K_{3,3}$ minor.

7. Let us say that a graph is *ci-plane* if it satisfies the definition of a plane graph with the exception that the sets A are not polygonal arcs, but continuous images of $[0,1]$ instead. Prove that every ci-plane graph is planar.

8. Let us say that a graph is *cs-plane* if it satisfies the definition of a plane graph with the exception that the sets A are not polygonal arcs, but connected subsets of the plane instead. Prove that every graph is isomorphic to a cs-plane graph. (Recall that a topological space X is connected if it has no nonempty proper subset that is both open and closed.)
Hint. The set consisting of the origin and all points with coordinates $(x, \sin(1/x))$ for $x > 0$ is connected.