

## WEEK 4 PROBLEMS

Math 6014A

1. Let  $k \geq 2$  be an integer. Prove that if a graph  $G$  is  $k$ -connected, and  $v_1, v_2, \dots, v_k$  are vertices of  $G$ , then there is a cycle in  $G$  that contains all  $v_i$ ,  $1 \leq i \leq k$ .

*Hint.* Use induction to get a cycle through  $v_1, v_2, \dots, v_{k-1}$ .

2. Prove that if  $G$  is 3-regular, then the vertex-connectivity equals the edge-connectivity.

3. Let  $k \geq 2$  be an integer. Prove that every connected  $k$ -regular bipartite multigraph on at least three vertices is 2-connected.

4. Let  $k > 1$  be an integer. Prove that every  $k$ -connected graph on at least  $2k$  vertices has a cycle of length at least  $2k$ .

5. Let  $G$  be a 2-connected graph, and let  $s, t$  be distinct vertices of  $G$ . Prove that the vertices of  $G$  can be numbered  $v_1, v_2, \dots, v_n$  in such a way that  $v_1 = s$ ,  $v_n = t$  and for all  $i = 2, 3, \dots, n$  the vertex  $v_i$  has a neighbor in  $\{v_1, v_2, \dots, v_{i-1}\}$  and the vertex  $v_{i-1}$  has a neighbor in  $\{v_i, v_{i+1}, \dots, v_n\}$ . (This is called an  $s$ - $t$  numbering.)

6. Let  $G$  be a 2-connected graph on at least four vertices and let  $e$  be an edge of  $G$ . Prove that either  $G \setminus e$  or  $G/e$  is 2-connected.